Pushing Astrometry to the Limit

Richard Berry

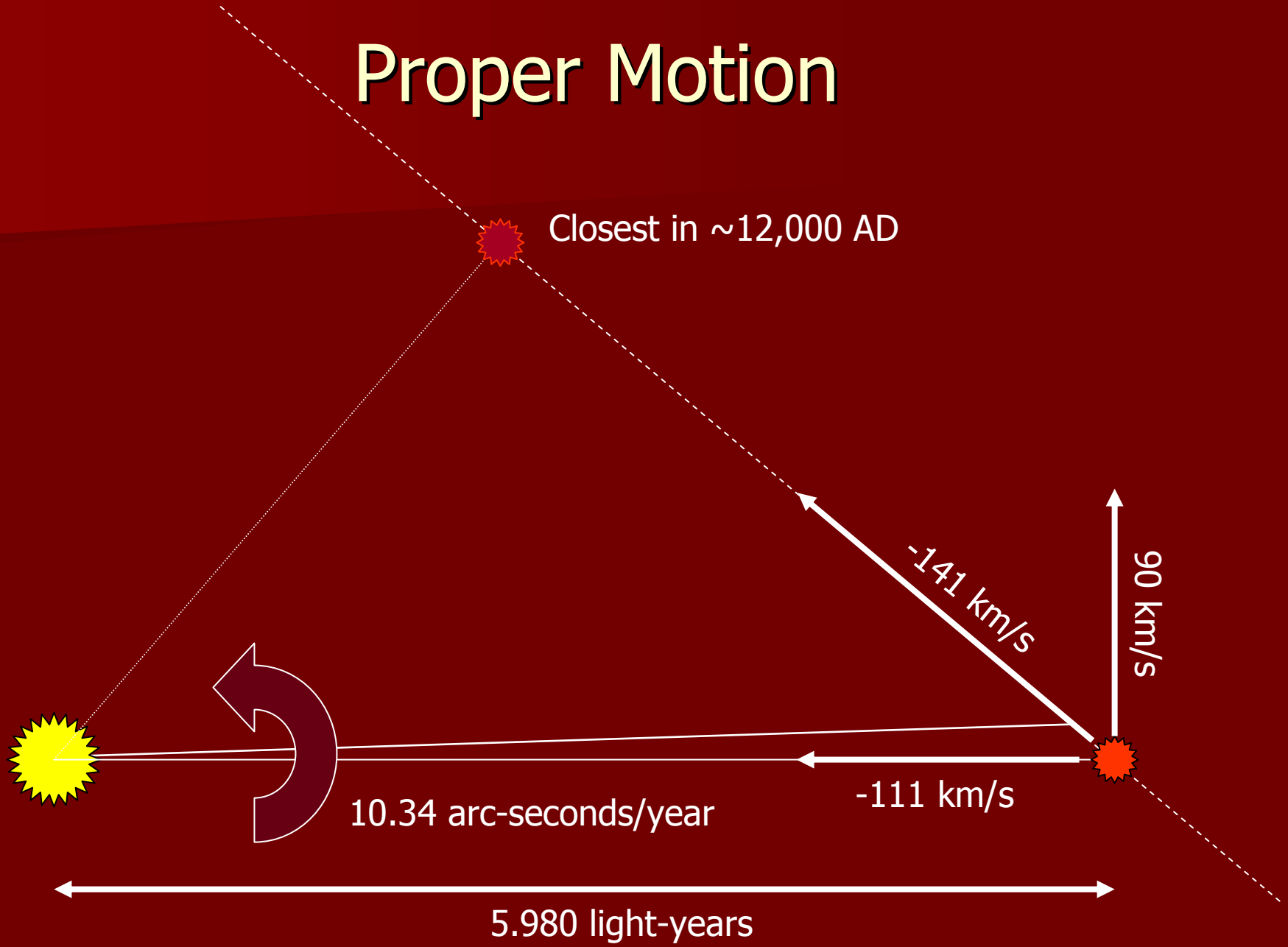
April 2010

NEAIC

Barnard's Star

- Location: Ophiuchus
- Coordinates: $17^h57^m48.5$ +4º41'36"(J2000)
- Apparent Magnitude: V = 9.54 (variable)
- Spectral Class: M5V (red dwarf)
- Proper Motion: 10.33777 arc-seconds/year
- Parallax: 0.5454 arc-seconds
- Distance: 5.980±0.003 light-years
- Radial Velocity: –110.6 km/second
- Rotation Period: 130.4 days

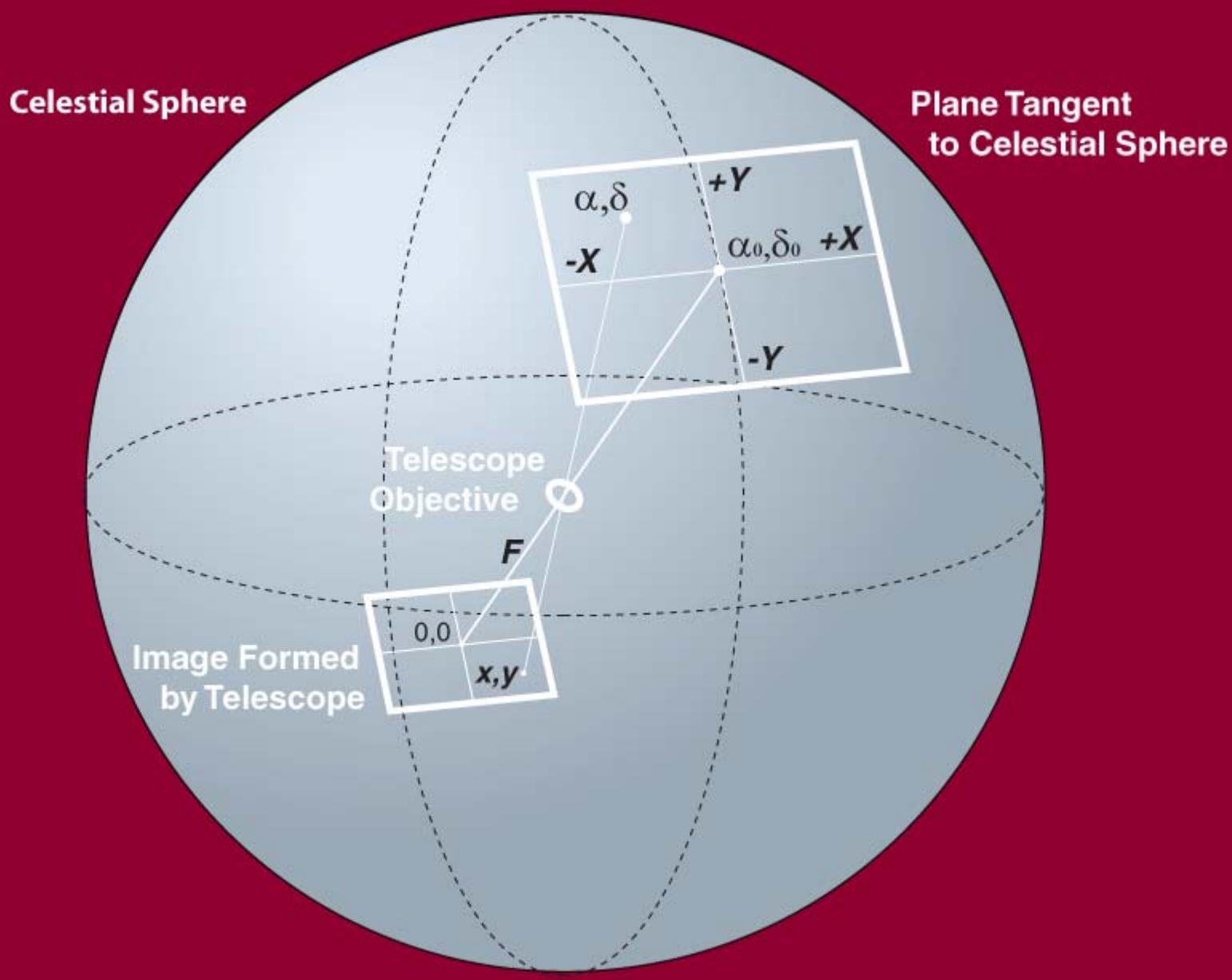
Proper Motion




N
Barnard's "Flying Star"
E
W
S

Astrometry with CCDs

- Requirements:
  - An image showing the object to be measured.
  - At least three reference stars in the image.
  - An astrometric catalog of reference stars (UCAC2).
  - Approximate coordinates for the image.
  - Software written for doing astrometry.
- Data also needed:
  - Observer's latitude, longitude, and time zone.
  - The dates and times images were made.


Celestial Sphere
Plane Tangent
to Celestial Sphere
α,δ
+Y
-X
α₀,δ₀
+X
-Y
Telescope
Objective
F
0,0
x,y
Image Formed
by Telescope

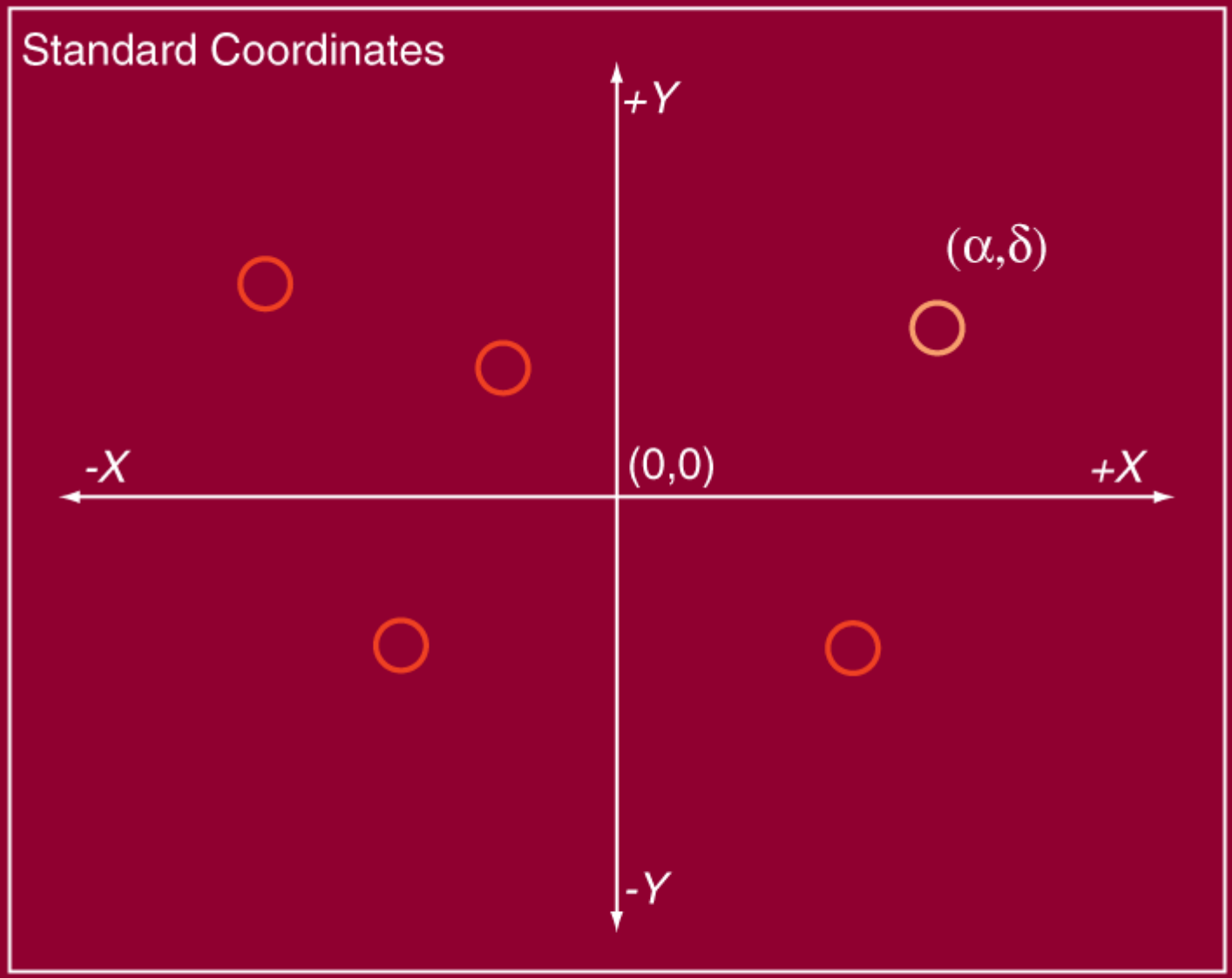
$(\alpha,\delta) \rightarrow (X,Y)$

- When you shoot an image, you're mapping the celestial spherical onto a plane surface.
- This occurs for all the stars in the image, both the target stars and the reference stars.
- The standard (X,Y) coordinates of a star at (α,δ) for an image centered on (α_0,δ_0) are:

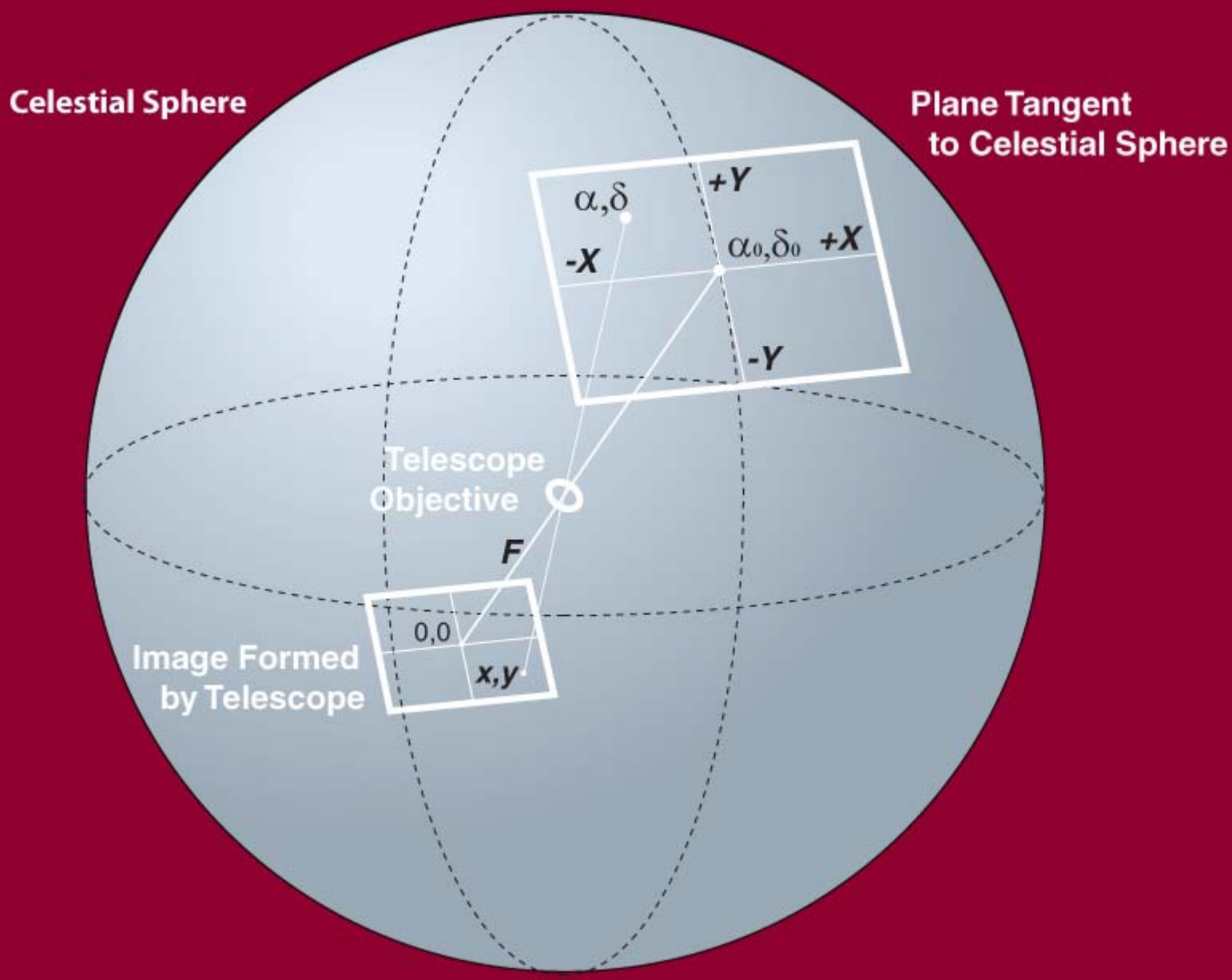
$$X = (\cos\delta \sin(\alpha-\alpha_0))/d$$

$$Y = (\sin\delta_0 \cos\delta \cos(\alpha-\alpha_0) - \cos\delta_0 \sin\delta)/d$$

where $d = \cos\delta_0 \cos\delta \cos(\alpha-\alpha_0)+\sin\delta_0 \sin\delta$.



This represents a plane tangent to the sky.
Each star at some (α,δ) has standard coordinates (*X*, *Y*).


Celestial Sphere
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α,δ
+Y
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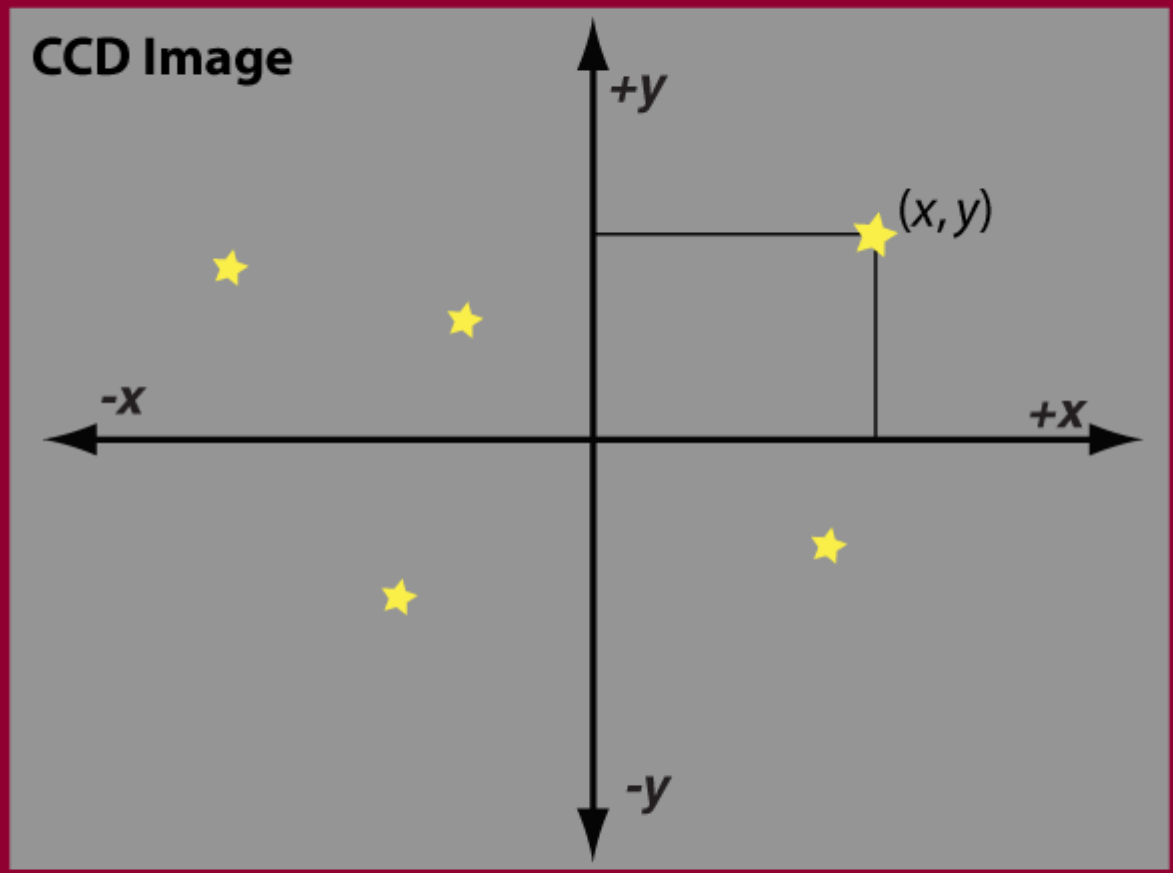
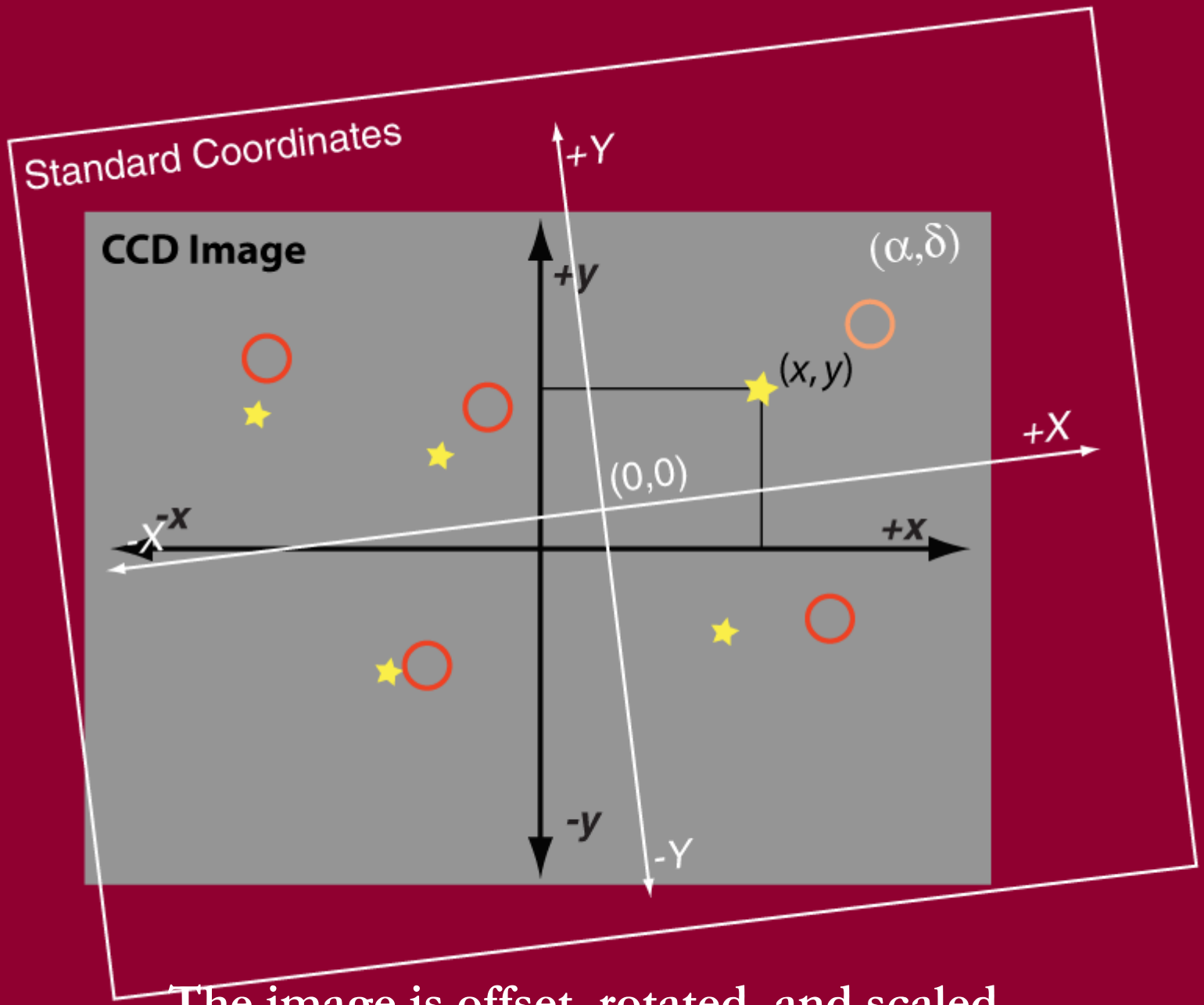
Telescope
Objective
F
0,0
x,y
Image Formed
by Telescope



This represents an image captured by a CCD camera. Each star in the image has a location (*x*, *y*).

The CCD Image

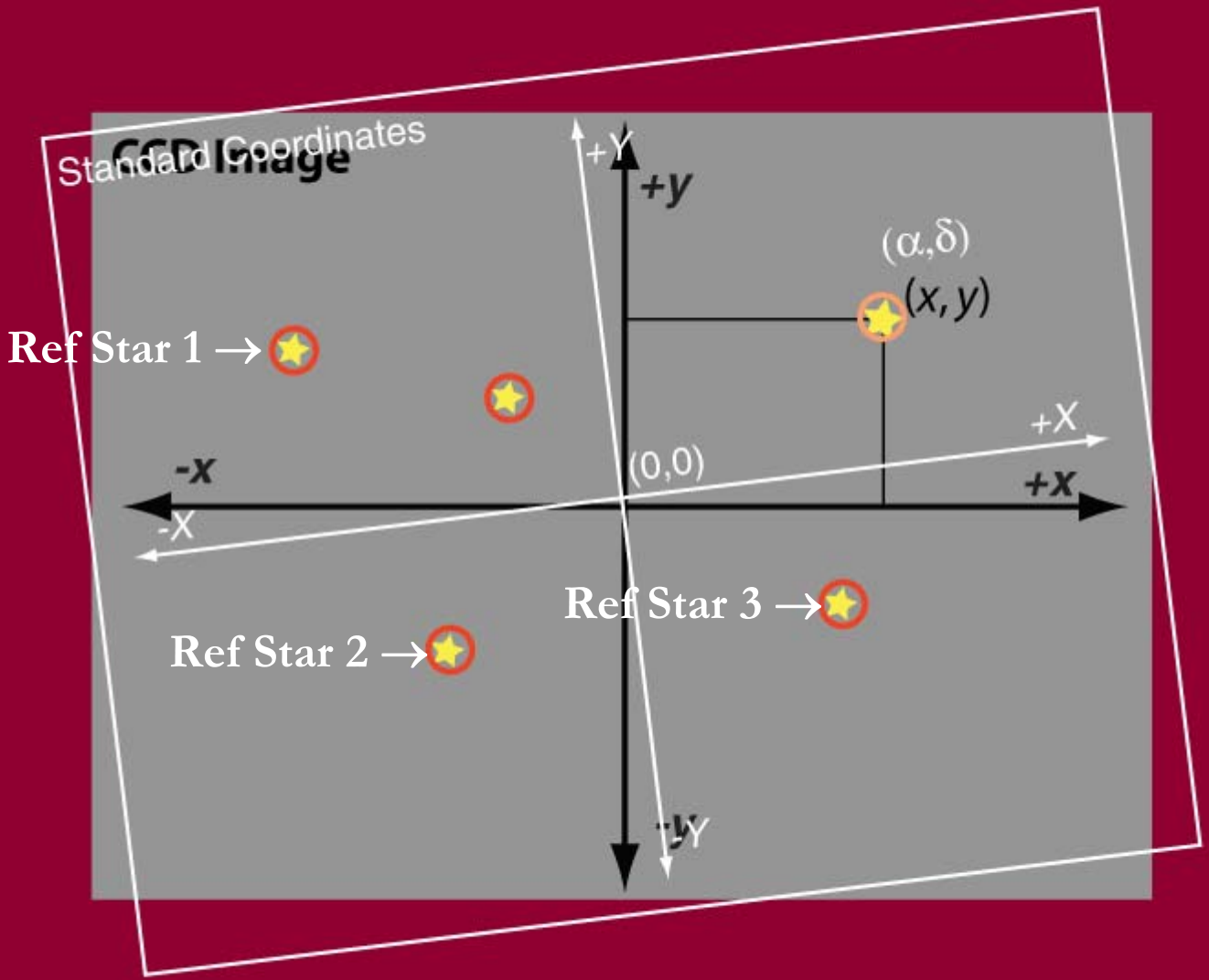
- Known properties of the image:
  - Approximate center coordinates: (α_0,δ_0).
  - Approximate focal length of telescope = F.
- Unknown properties of the image:
  - Offset distance in x axis: x_{offset}.
  - Offset distance in y axis: y_{offset}.
  - Rotation relative to north-at-top = ρ.



The image is offset, rotated, and scaled with respect to standard coordinates.

Reference stars

- Astrometric catalogs are lists of stars with accurately measured (α,δ) coordinates.
  - Guide Star Catalog (GSC)
  - USNO A2.0
  - UCAC2 or UCAC3
- Astrometric catalogs often list ***millions*** of stars.
- We use the reference stars in the image to link image coordinates to standard coordinates.
- A minimum of three reference stars are needed.



By offsetting, rotating, and scaling standard coordinates, we can link each reference star with its counterpart in the image.

What do we know?

- We know:
  - Three or more reference stars in the image.
  - Approximate coordinates of image center (α_0,δ_0).
  - For each reference star, its (α,δ) coordinates.
  - For each reference, its standard coordinates (X, Y).
  - For each reference, we measure (x,y) from the image.
  - For target object(s), we measure (x,y) coordinates.
- We want:
  - The (α,δ) coordinates of the target object.

$(x,y) \rightarrow (X,Y)$

- To offset, rotate, and scale coordinates:
  - $X = x \cos\rho/F + y \sin\rho/F + x_{offset}/F$
  - $Y = x \sin\rho/F + y \cos\rho/F + y_{offset}/F$
- But we do not know ρ, F, or the offsets.
- However, for each reference star, we know:
  - (X,Y) standard coordinates, and
  - (x,y) image coordinates.

Linking the Coordinates

- Suppose we have three reference stars.
- For each star, we know (x,y) and (X,Y).
  - $X_1 = ax_1 + by_1 + c$ and $Y_1 = dx_1 + dy_1 + f$
  - $X_2 = ax_2 + by_2 + c$ and $Y_2 = dx_2 + dy_2 + f$
  - $X_3 = ax_3 + by_3 + c$ and $Y_3 = dx_3 + dy_3 + f$.
- Three equations, three unknowns $\rightarrow$ solvable.
- In the X axis, we solve for *a*, *b*, and *c*.
- In the Y axis, we solve for *d*, *e*, and *f*.

Computing Target Coordinates

- From reference stars, we find *a, b, c, d, e,* and *f*.
- The standard coordinates of the target are:
  - $X_{target} = ax_{target} + by_{target} + c$, and
  - $Y_{target} = dx_{target} + ey_{target} + f$
- Given (X, Y) for the target, it's (α,δ) is:
  - $\delta = \arcsin((\sin\delta_0 + Y\cos\delta_0)/(\sqrt{1+X^2+Y^2}))$, and
  - $\alpha = \alpha_0 + \arctan(X/(\cos\delta_0 + Y\sin\delta_0))$.

- Ta-da!

Parallax: Mission ~~Impossible~~ Difficult

- Goals:
  - Repeatedly measure α and δ for a year.
  - Attain accuracy ~1% the expected parallax.
  - Reduce and analyze the measurements.
- Problems to overcome:
  - Differential refraction displacing stars.
  - Instrumental effects of all kinds.
  - Under- and over-exposure effects.
  - Errors and proper motion in reference stars.

Shooting Images

- When to shoot
  - If possible, near the meridian.
  - If possible, on nights with good seeing.
  - If possible, once a week, more often when star 90º from Sun.
- Filters
  - To minimize differential refraction, use V or R.
- Reference stars
  - Select reference stars with low proper motion.
  - Set exposure time for high signal-to-noise ratio.
- Target star
  - Do not allow image to reach saturation.
- How many images?
  - Shoot as many as practical to shoot and reduce.

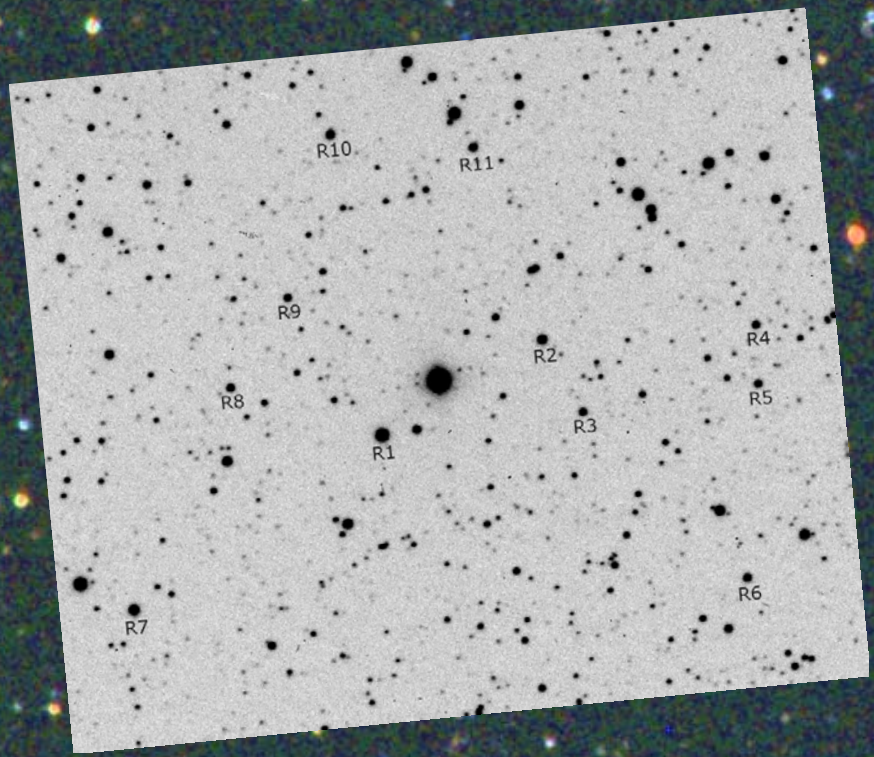
Extracting Coordinates

- In AIP4Win, semi-automated process
  - Observer must exercise oversight.
  - Check/verify all ingoing parameters.
  - Select an optimum set of reference stars.
  - Supervise extraction and processing.
  - Inspect reported data.
  - Check discrepancies and anomalies.


N
E
W
S

N

Define a set of reference stars...

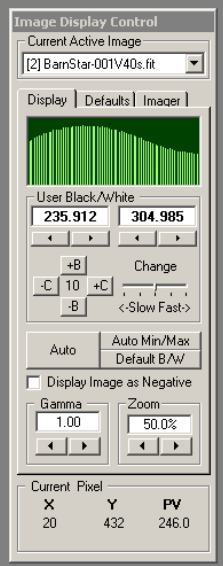


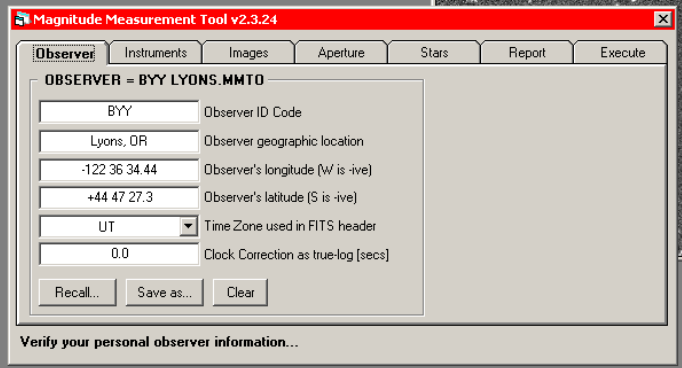
E

W

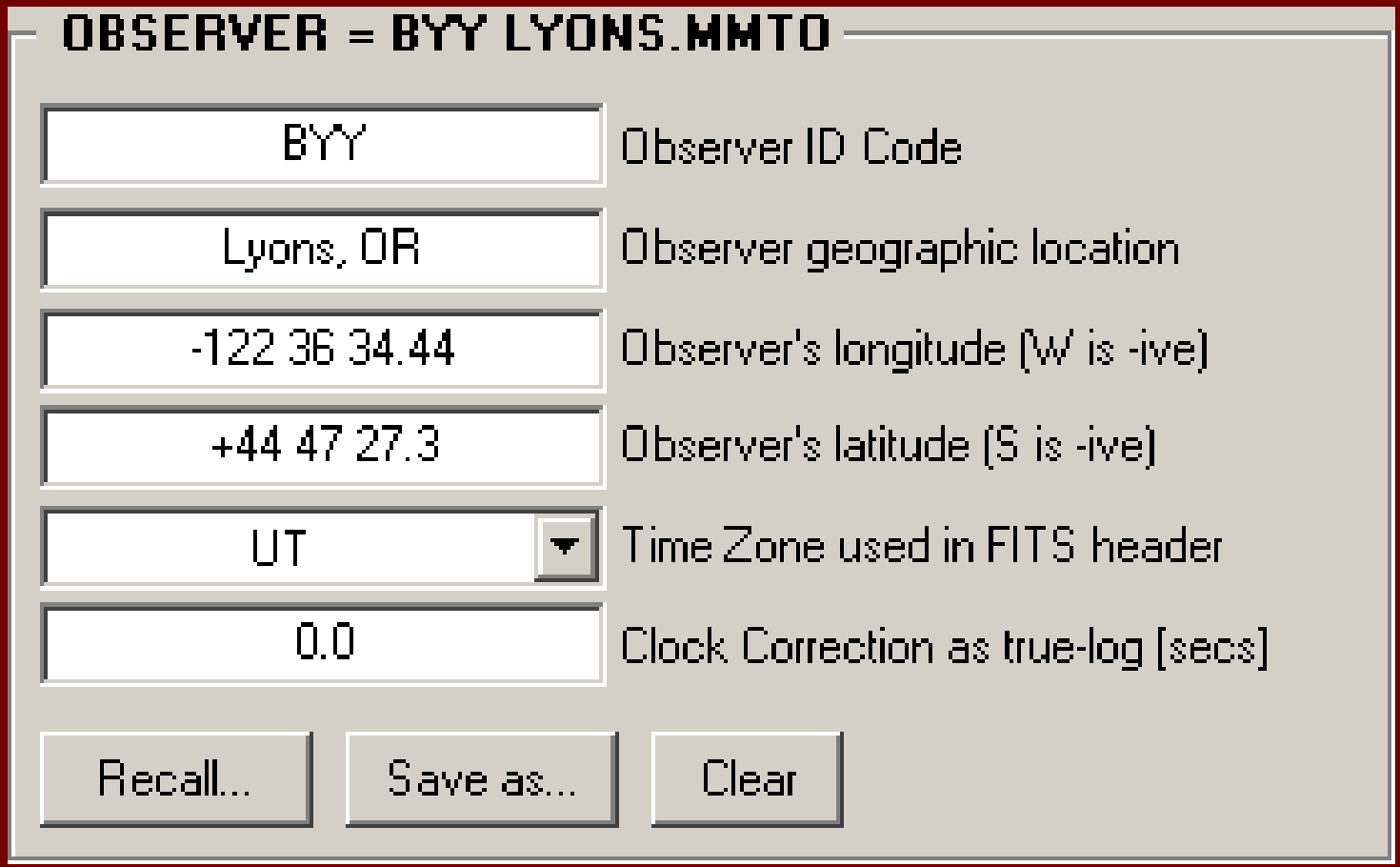
S

Getting started…





Observer Properties...



Instrument Properties…

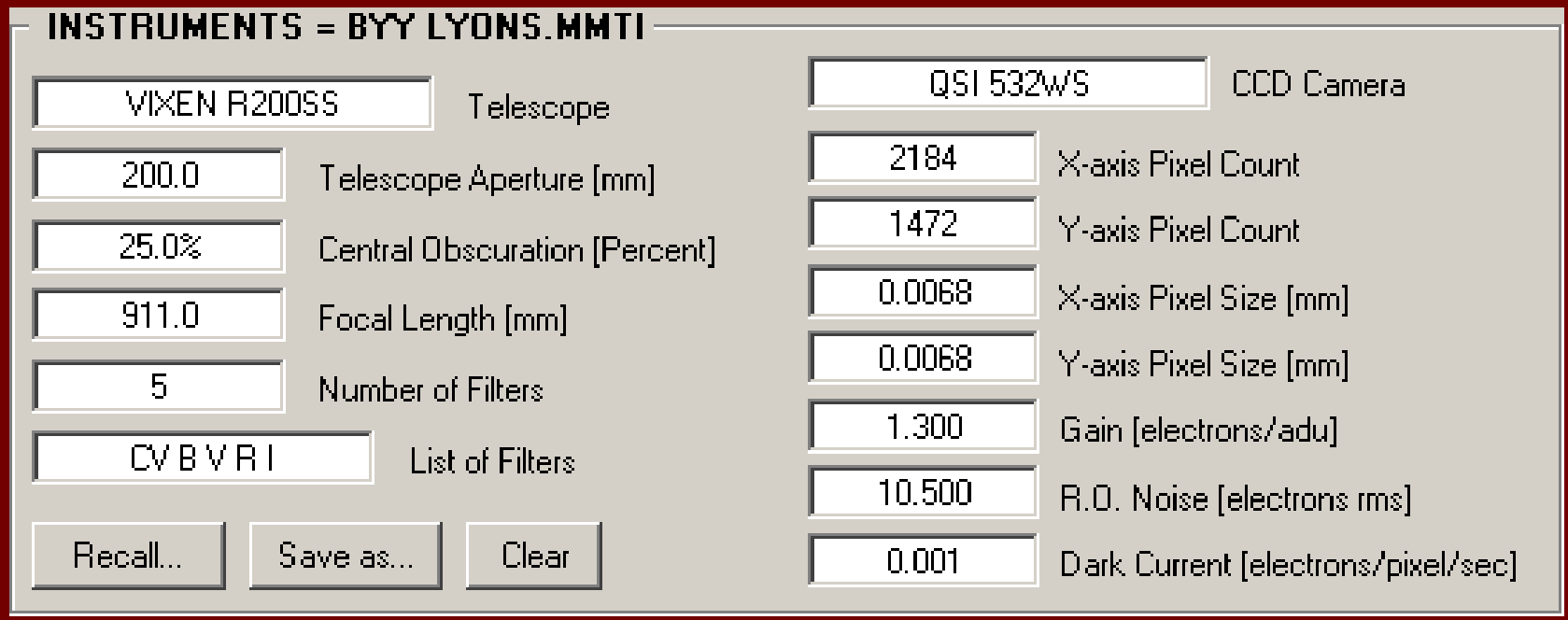
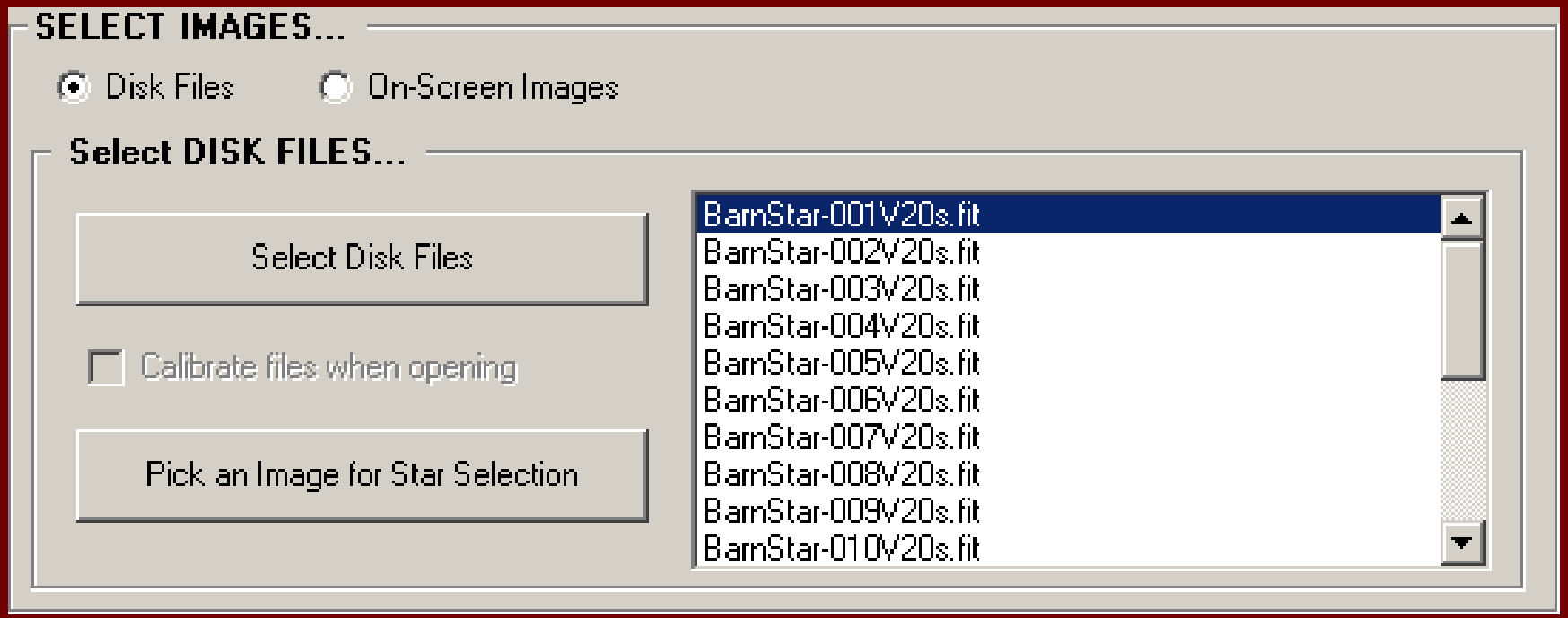
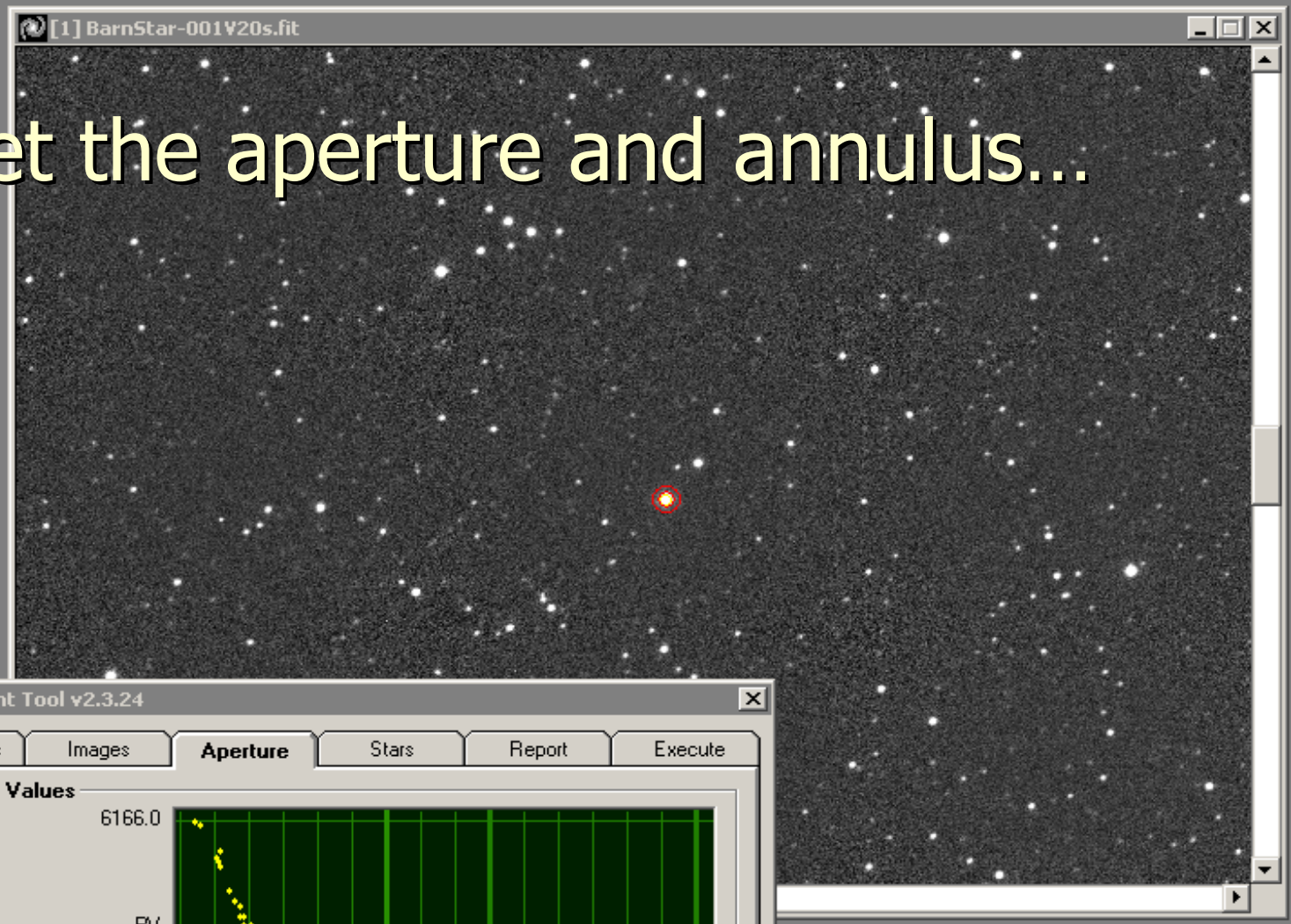


Image Selection…



Set the aperture and annulus...



Magnitude Measurement Tool v2.3.24

Observer | Instruments | Images | **Aperture** | Stars | Report | Execute

APERTURE = Default Values

6.0	Aperture
9.0	Inner Annulus
15.0	Outer Annulus
20.0	Zero Point

10.563 ±0.003

6166.0

PV

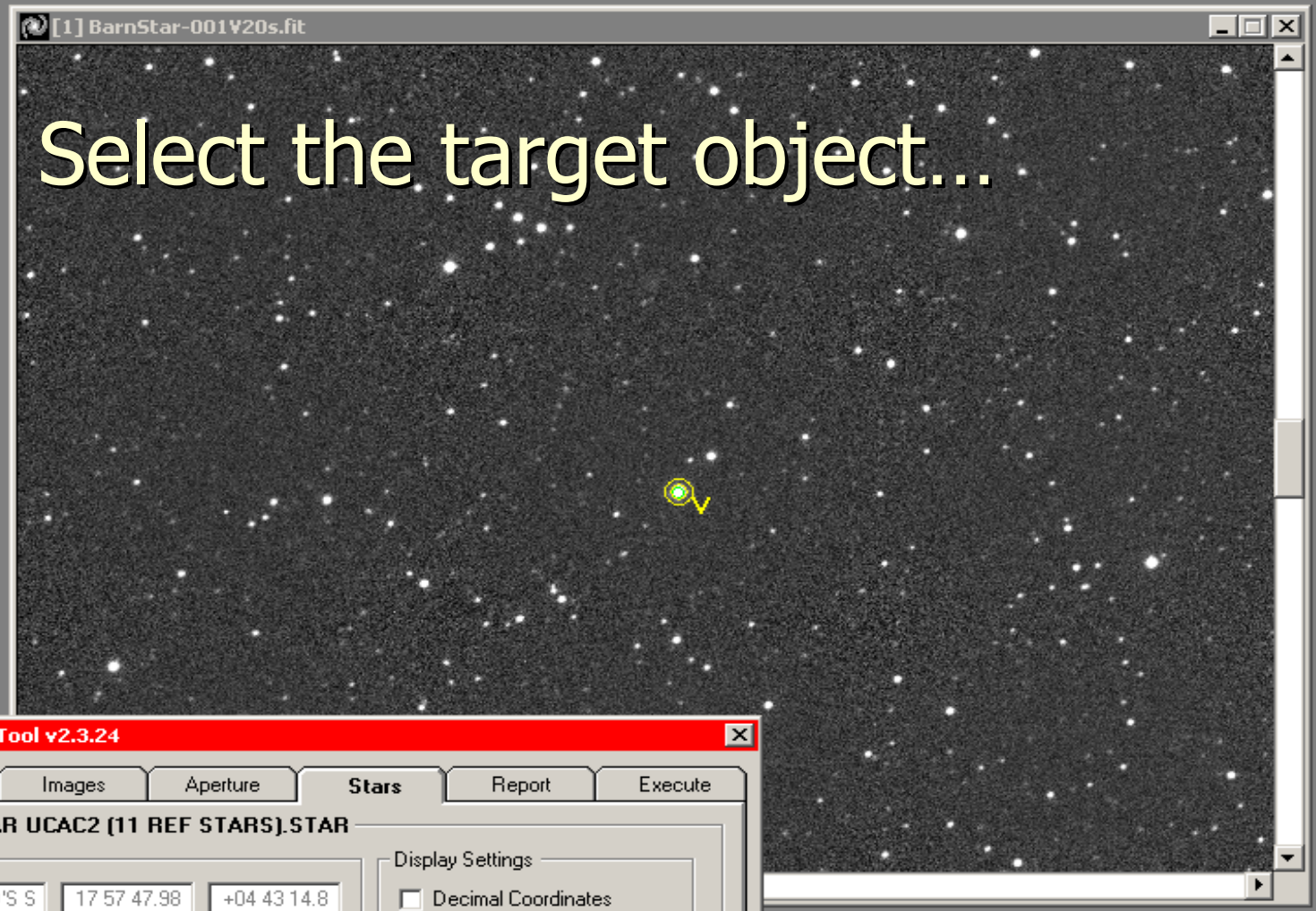
250.807

Aperture = 6.0 9.0 > Annulus > 15.0

Recall... Save as... Clear

(•) Profile ( ) Curve of Growth ( ) SNR

Optimize the photometry settings...



Magnitude Measurement Tool v2.3.24

Observer | Instruments | Images | Aperture | **Stars** | Report | Execute

STAR DATA = BARNSTAR UCAC2 (11 REF STARS).STAR

Target = Variable star

V	BARNARD'S S	17 57 47.98	+04 43 14.8

Display Settings

☐ Decimal Coordinates

Star 1 of 11

C1	R1	17 57 51.96	+04 42 20.2

Filter 1 of 4 | | 0.000 | ± | 0.000

Edit Tools

☐ Enable Star Editing

Add | Delete | Sort

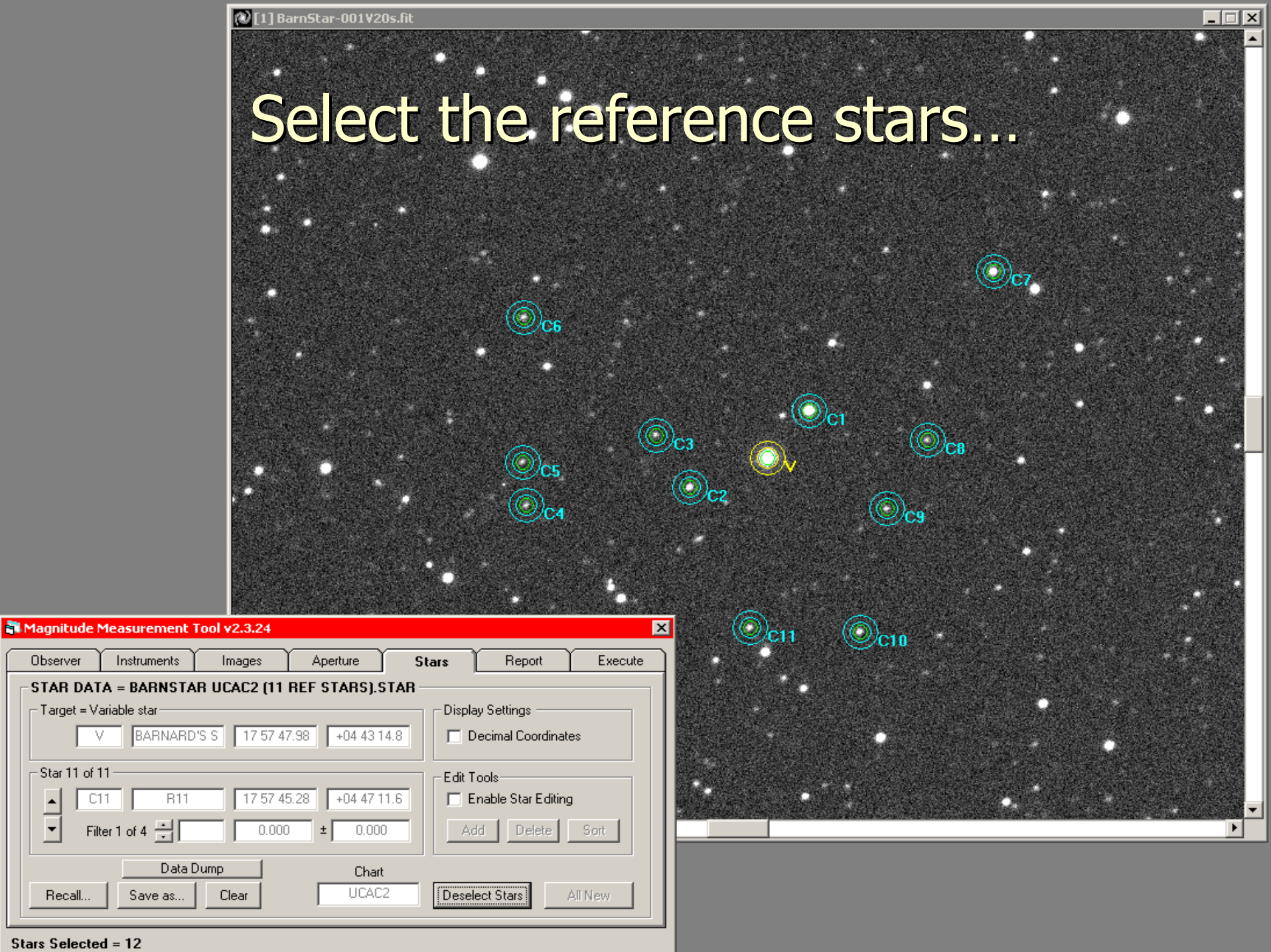
Data Dump

Recall... | Save as... | Clear

Chart: UCAC2

Deselect Stars | All New

Stars Selected = 1


[1] BarnStar-001V20s.fit
Select the reference stars...
C6
C7
C1
C3
C8
V
C5
C2
C4
C9
C11
C10
Magnitude Measurement Tool v2.3.24
Observer
Instruments
Images
Aperture
Stars
Report
Execute
STAR DATA = BARNSTAR UCAC2 (11 REF STARS).STAR
Target = Variable star
V
BARNARD'S S
17 57 47.98
+04 43 14.8
Display Settings
Decimal Coordinates
Star 11 of 11
C11
R11
17 57 45.28
+04 47 11.6
Filter 1 of 4
0.000
±
0.000
Edit Tools
Enable Star Editing
Add
Delete
Sort
Data Dump
Recall...
Save as...
Clear
Chart
UCAC2
Deselect Stars
All New
Stars Selected = 12

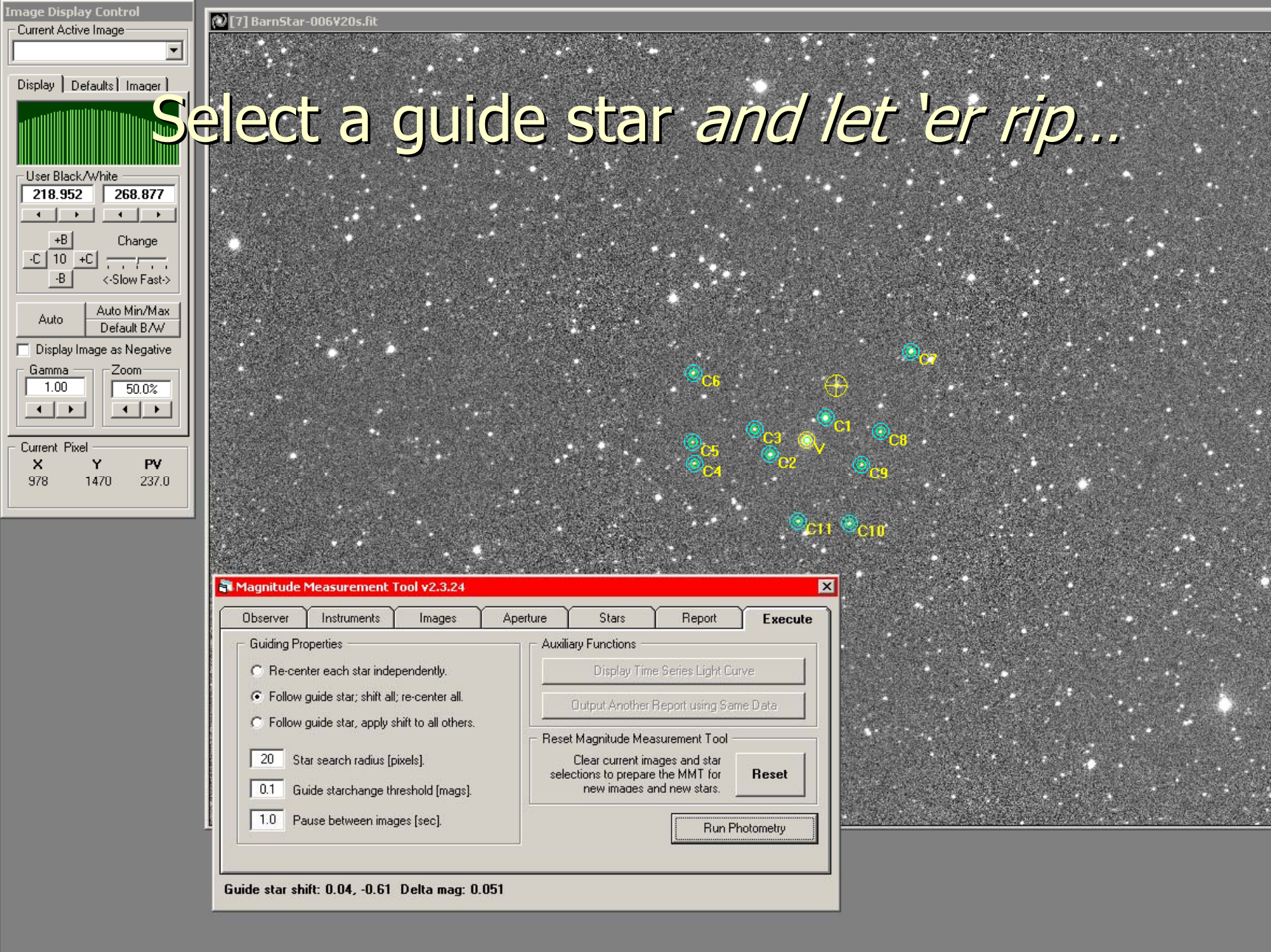
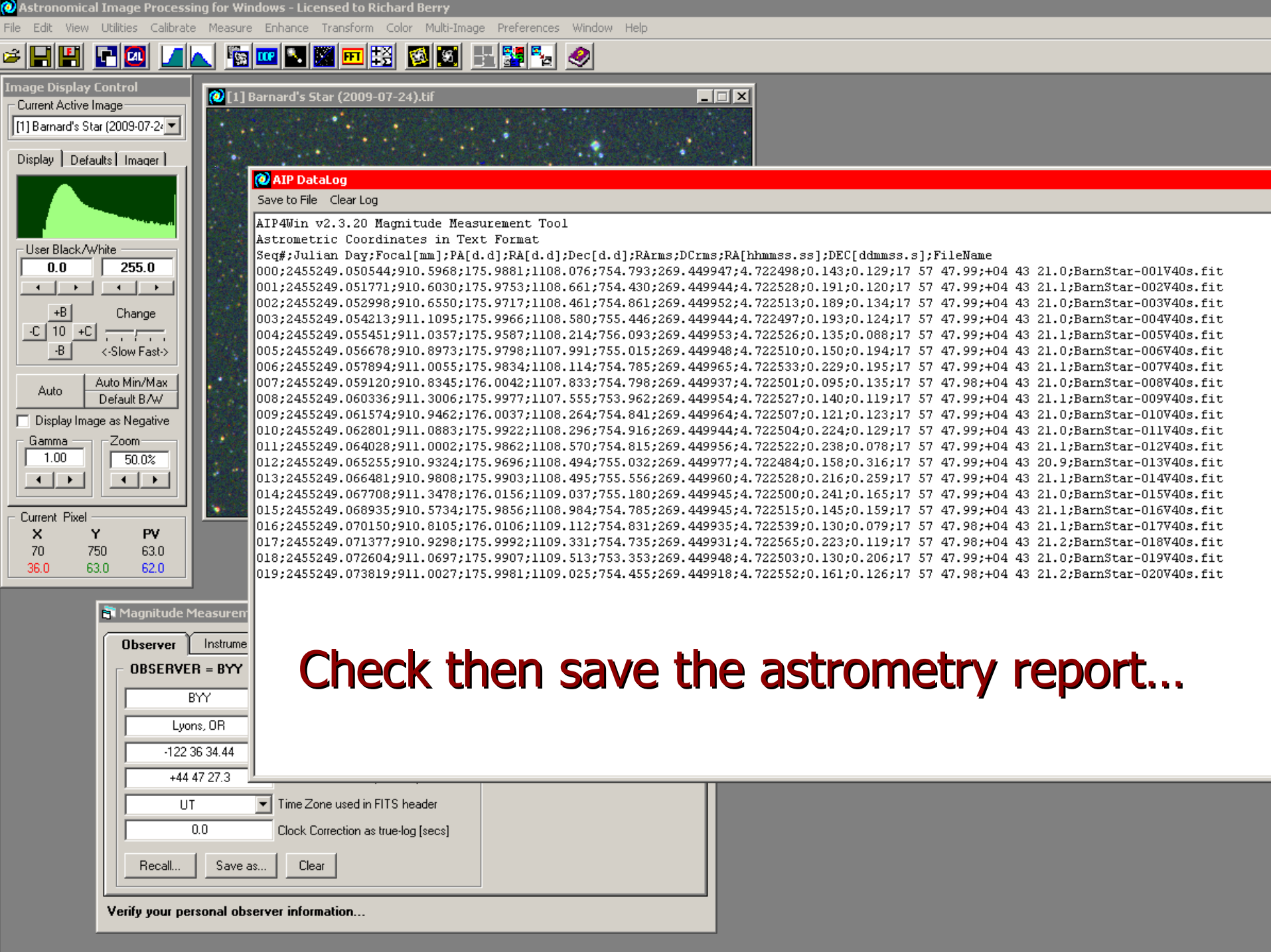


Image Display Control
Current Active Image
Display
Defaults
Imager
User Black/White
218.952
268.877
+B
-C
10
+C
-B
Change
<-Slow Fast->
Auto
Auto Min/Max
Default B/W
Display Image as Negative
Gamma
1.00
Zoom
50.0%
Current Pixel
X
Y
PV
978
1470
237.0
[7] BarnStar-006V20s.fit
Select a guide star and let 'er rip...
C6
C7
C1
C3
C8
C5
V
C2
C4
C9
C11
C10
Magnitude Measurement Tool v2.3.24
Observer
Instruments
Images
Aperture
Stars
Report
Execute
Guiding Properties
Re-center each star independently.
Follow guide star; shift all; re-center all.
Follow guide star, apply shift to all others.
20
Star search radius [pixels].
0.1
Guide starchange threshold [mags].
1.0
Pause between images [sec].
Auxiliary Functions
Display Time Series Light Curve
Output Another Report using Same Data
Reset Magnitude Measurement Tool
Clear current images and star selections to prepare the MMT for new images and new stars.
Reset
Run Photometry
Guide star shift: 0.04, -0.61 Delta mag: 0.051


Astronomical Image Processing for Windows - Licensed to Richard Berry
File Edit View Utilities Calibrate Measure Enhance Transform Color Multi-Image Preferences Window Help
Image Display Control
Current Active Image
[1] Barnard's Star (2009-07-24).tif
Display Defaults Imager
User Black/White
0.0
255.0
+B
-C 10 +C
-B
Change
<-Slow Fast->
Auto
Auto Min/Max
Default B/W
Display Image as Negative
Gamma
1.00
Zoom
50.0%
Current Pixel
X Y PV
70 750 63.0
36.0 63.0 62.0
[1] Barnard's Star (2009-07-24).tif
AIP DataLog
Save to File Clear Log
AIP4Win v2.3.20 Magnitude Measurement Tool
Astrometric Coordinates in Text Format
Seq#;Julian Day;Focal[mm];PA[d.d];RA[d.d];Dec[d.d];RArms;DCrms;RA[hhmmss.ss];DEC[ddmmss.s];FileName
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001;2455249.051771;910.6030;175.9753;1108.661;754.430;269.449944;4.722528;0.191;0.120;17 57 47.99;+04 43 21.1;BarnStar-002V40s.fit
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004;2455249.055451;911.0357;175.9587;1108.214;756.093;269.449953;4.722526;0.135;0.088;17 57 47.99;+04 43 21.1;BarnStar-005V40s.fit
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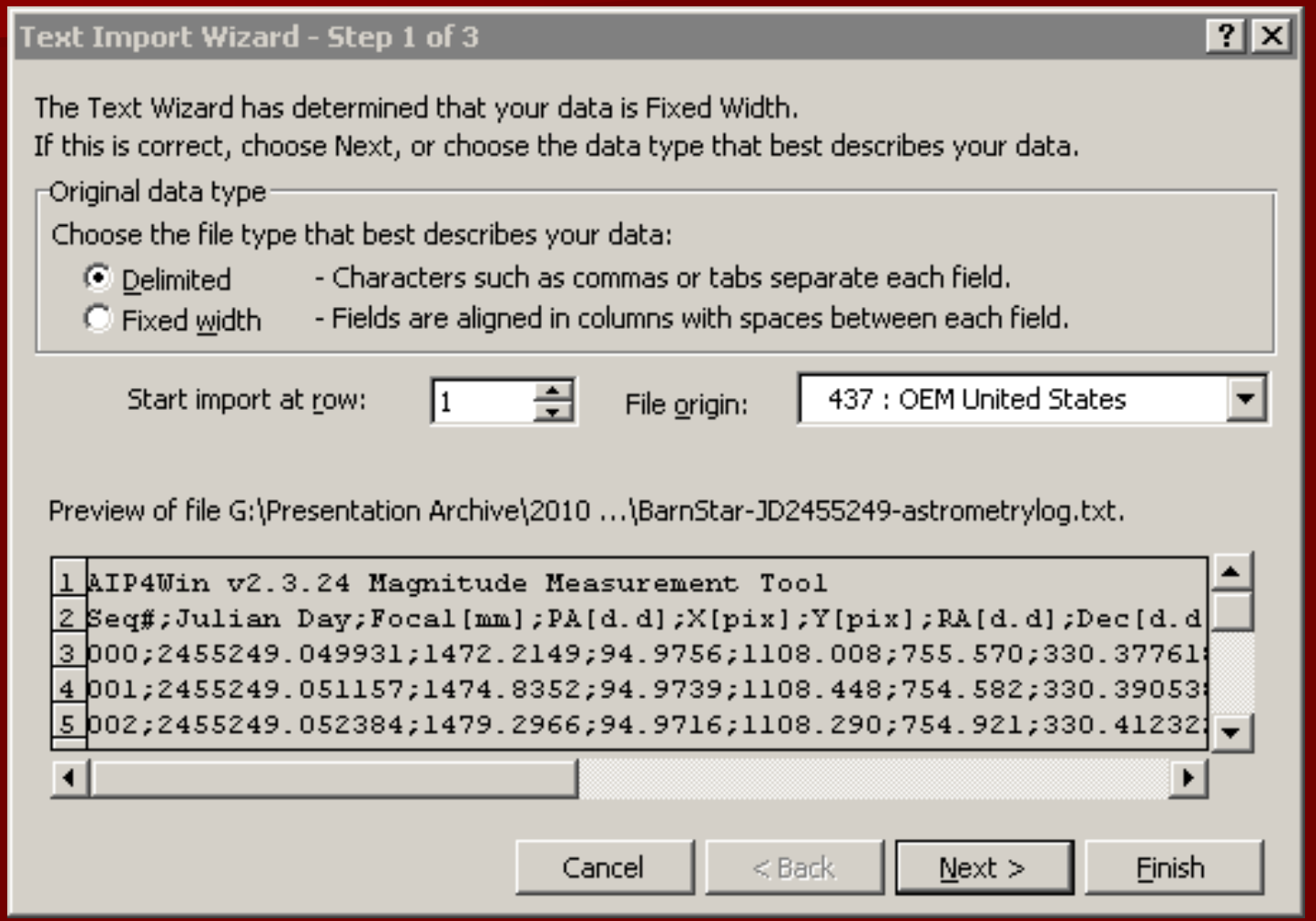
019;2455249.073819;911.0027;175.9981;1109.025;754.455;269.449918;4.722552;0.161;0.126;17 57 47.98;+04 43 21.2;BarnStar-020V40s.fit
Magnitude Measuren
Observer
Instrume
OBSERVER = BYY
BYY
Lyons, OR
-122 36 34.44
+44 47 27.3
UT
Time Zone used in FITS header
0.0
Clock Correction as true-log [secs]
Recall...
Save as...
Clear
Verify your personal observer information...
Check then save the astrometry report...

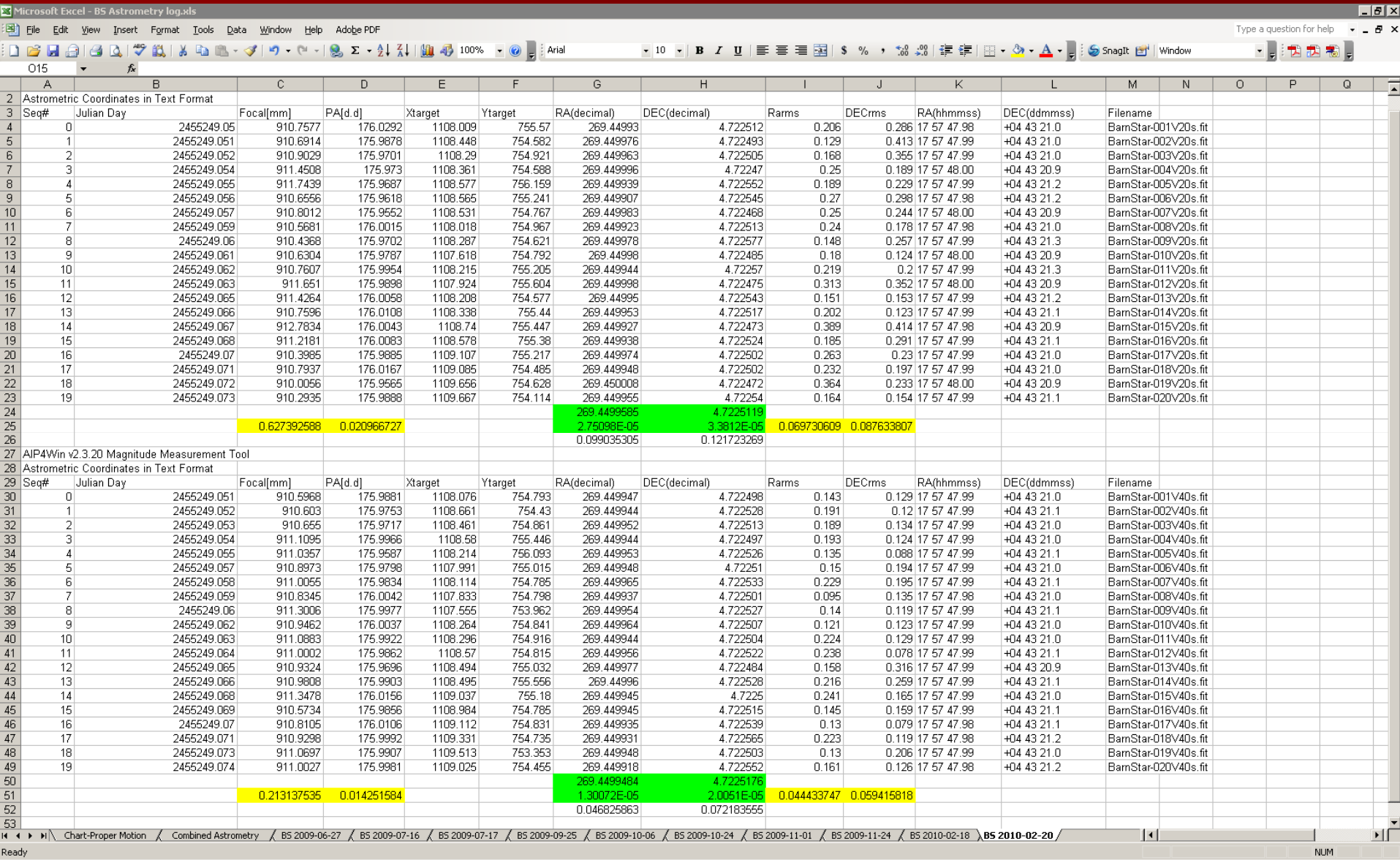
Examination

- Copy data to Excel (or other spreadsheet)
  - Importing is easy when text data is delimited.
  - Check the "canaries": focal length, position angle.
  - Check the residuals in α and δ.
  - Compute (α,δ) mean and standard deviation.
  - Plot individual and mean positions.
- Long term
  - Plot the individual and mean positions for all nights.
  - Apply lessons learned to future observations.

Importing a text file…



Astrometry data imported into Excel...



Microsoft Excel - BS Astrometry log.xls

File Edit View Insert Format Tools Data Window Help Adobe PDF

O15

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
2	Astrometric Coordinates in Text Format																
3	Seq#	Julian Day	Focal[mm]	PA[d.d]	Xtarget	Ytarget	RA(decimal)	DEC(decimal)	Rarms	DECrms	RA(hhmmss)	DEC(ddmmss)	Filename				
4	0	2455249.05	910.7577	176.0292	1108.009	755.57	269.44993	4.722512	0.206	0.286	17 57 47.98	+04 43 21.0	BarnStar-001V20s.fit				
5	1	2455249.051	910.6914	175.9878	1108.448	754.582	269.449976	4.722493	0.129	0.413	17 57 47.99	+04 43 21.0	BarnStar-002V20s.fit				
6	2	2455249.052	910.9029	175.9701	1108.29	754.921	269.449963	4.722505	0.168	0.355	17 57 47.99	+04 43 21.0	BarnStar-003V20s.fit				
7	3	2455249.054	911.4508	175.973	1108.361	754.588	269.449996	4.72247	0.25	0.189	17 57 48.00	+04 43 20.9	BarnStar-004V20s.fit				
8	4	2455249.055	911.7439	175.9687	1108.577	756.159	269.449939	4.722552	0.189	0.229	17 57 47.99	+04 43 21.2	BarnStar-005V20s.fit				
9	5	2455249.056	910.6556	175.9618	1108.565	755.241	269.449907	4.722545	0.27	0.298	17 57 47.98	+04 43 21.2	BarnStar-006V20s.fit				
10	6	2455249.057	910.8012	175.9552	1108.531	754.767	269.449983	4.722468	0.25	0.244	17 57 48.00	+04 43 20.9	BarnStar-007V20s.fit				
11	7	2455249.059	910.5681	176.0015	1108.018	754.967	269.449923	4.722513	0.24	0.178	17 57 47.98	+04 43 21.0	BarnStar-008V20s.fit				
12	8	2455249.06	910.4368	175.9702	1108.287	754.621	269.449978	4.722577	0.148	0.257	17 57 47.99	+04 43 21.3	BarnStar-009V20s.fit				
13	9	2455249.061	910.6304	175.9787	1107.618	754.792	269.44998	4.722485	0.18	0.124	17 57 48.00	+04 43 20.9	BarnStar-010V20s.fit				
14	10	2455249.062	910.7607	175.9954	1108.215	755.205	269.449944	4.72257	0.219	0.2	17 57 47.99	+04 43 21.3	BarnStar-011V20s.fit				
15	11	2455249.063	911.651	175.9898	1107.924	755.604	269.449998	4.722475	0.313	0.352	17 57 48.00	+04 43 20.9	BarnStar-012V20s.fit				
16	12	2455249.065	911.4264	176.0058	1108.208	754.577	269.44995	4.722543	0.151	0.153	17 57 47.99	+04 43 21.2	BarnStar-013V20s.fit				
17	13	2455249.066	910.7596	176.0108	1108.338	755.44	269.449953	4.722517	0.202	0.123	17 57 47.99	+04 43 21.1	BarnStar-014V20s.fit				
18	14	2455249.067	912.7834	176.0043	1108.74	755.447	269.449927	4.722473	0.389	0.414	17 57 47.98	+04 43 20.9	BarnStar-015V20s.fit				
19	15	2455249.068	911.2181	176.0083	1108.578	755.38	269.449938	4.722524	0.185	0.291	17 57 47.99	+04 43 21.1	BarnStar-016V20s.fit				
20	16	2455249.07	910.3985	175.9885	1109.107	755.217	269.449974	4.722502	0.263	0.23	17 57 47.99	+04 43 21.0	BarnStar-017V20s.fit				
21	17	2455249.071	910.7937	176.0167	1109.085	754.485	269.449948	4.722502	0.232	0.197	17 57 47.99	+04 43 21.0	BarnStar-018V20s.fit				
22	18	2455249.072	910.0056	175.9565	1109.656	754.628	269.450008	4.722472	0.364	0.233	17 57 48.00	+04 43 20.9	BarnStar-019V20s.fit				
23	19	2455249.073	910.2935	175.9888	1109.667	754.114	269.449955	4.72254	0.164	0.154	17 57 47.99	+04 43 21.1	BarnStar-020V20s.fit				
24							269.4499585	4.7225119									
25			0.627392588	0.020966727			2.75098E-05	3.3812E-05	0.069730609	0.087633807							
26							0.099035305	0.121723269									
27	AIP4Win v2.3.20 Magnitude Measurement Tool																
28	Astrometric Coordinates in Text Format																
29	Seq#	Julian Day	Focal[mm]	PA[d.d]	Xtarget	Ytarget	RA(decimal)	DEC(decimal)	Rarms	DECrms	RA(hhmmss)	DEC(ddmmss)	Filename				
30	0	2455249.051	910.5968	175.9881	1108.076	754.793	269.449947	4.722498	0.143	0.129	17 57 47.99	+04 43 21.0	BarnStar-001V40s.fit				
31	1	2455249.052	910.603	175.9753	1108.661	754.43	269.449944	4.722528	0.191	0.12	17 57 47.99	+04 43 21.1	BarnStar-002V40s.fit				
32	2	2455249.053	910.655	175.9717	1108.461	754.861	269.449962	4.722513	0.189	0.134	17 57 47.99	+04 43 21.0	BarnStar-003V40s.fit				
33	3	2455249.054	911.1095	175.9966	1108.58	755.446	269.449944	4.722497	0.193	0.124	17 57 47.99	+04 43 21.0	BarnStar-004V40s.fit				
34	4	2455249.055	911.0357	175.9587	1108.214	756.093	269.449953	4.722526	0.135	0.088	17 57 47.99	+04 43 21.1	BarnStar-005V40s.fit				
35	5	2455249.057	910.8973	175.9798	1107.991	755.015	269.449948	4.72251	0.15	0.194	17 57 47.99	+04 43 21.0	BarnStar-006V40s.fit				
36	6	2455249.058	911.0055	175.9834	1108.114	754.785	269.449965	4.722533	0.229	0.195	17 57 47.99	+04 43 21.1	BarnStar-007V40s.fit				
37	7	2455249.059	910.8345	176.0042	1107.833	754.798	269.449937	4.722501	0.095	0.135	17 57 47.98	+04 43 21.0	BarnStar-008V40s.fit				
38	8	2455249.06	911.3006	175.9977	1107.555	753.962	269.449954	4.722527	0.14	0.119	17 57 47.99	+04 43 21.1	BarnStar-009V40s.fit				
39	9	2455249.062	910.9462	176.0037	1108.264	754.841	269.449964	4.722507	0.121	0.123	17 57 47.99	+04 43 21.0	BarnStar-010V40s.fit				
40	10	2455249.063	911.0883	175.9922	1108.296	754.916	269.449944	4.722504	0.224	0.129	17 57 47.99	+04 43 21.0	BarnStar-011V40s.fit				
41	11	2455249.064	911.0002	175.9862	1108.57	754.815	269.449956	4.722522	0.238	0.078	17 57 47.99	+04 43 21.1	BarnStar-012V40s.fit				
42	12	2455249.065	910.9324	175.9696	1108.494	755.032	269.449977	4.722484	0.158	0.316	17 57 47.99	+04 43 20.9	BarnStar-013V40s.fit				
43	13	2455249.066	910.9808	175.9903	1108.495	755.556	269.44996	4.722528	0.216	0.259	17 57 47.99	+04 43 21.1	BarnStar-014V40s.fit				
44	14	2455249.068	911.3478	176.0156	1109.037	755.18	269.449945	4.7225	0.241	0.165	17 57 47.99	+04 43 21.0	BarnStar-015V40s.fit				
45	15	2455249.069	910.5734	175.9856	1108.984	754.785	269.449945	4.722515	0.145	0.159	17 57 47.99	+04 43 21.1	BarnStar-016V40s.fit				
46	16	2455249.07	910.8105	176.0106	1109.112	754.831	269.449935	4.722539	0.13	0.079	17 57 47.98	+04 43 21.1	BarnStar-017V40s.fit				
47	17	2455249.071	910.9298	175.9992	1109.331	754.735	269.449931	4.722565	0.223	0.119	17 57 47.98	+04 43 21.2	BarnStar-018V40s.fit				
48	18	2455249.073	911.0697	175.9907	1109.513	753.353	269.449948	4.722503	0.13	0.206	17 57 47.99	+04 43 21.0	BarnStar-019V40s.fit				
49	19	2455249.074	911.0027	175.9981	1109.025	754.455	269.449918	4.722552	0.161	0.126	17 57 47.98	+04 43 21.2	BarnStar-020V40s.fit				
50							269.4499484	4.7225176									
51			0.213137535	0.014251584			1.30072E-05	2.0051E-05	0.044433747	0.059415818							
52							0.046825863	0.072183555									
53																	

Chart-Proper Motion | Combined Astrometry | BS 2009-06-27 | BS 2009-07-16 | BS 2009-07-17 | BS 2009-09-25 | BS 2009-10-06 | BS 2009-10-24 | BS 2009-11-01 | BS 2009-11-24 | BS 2010-02-18 | **BS 2010-02-20**

Ready NUM

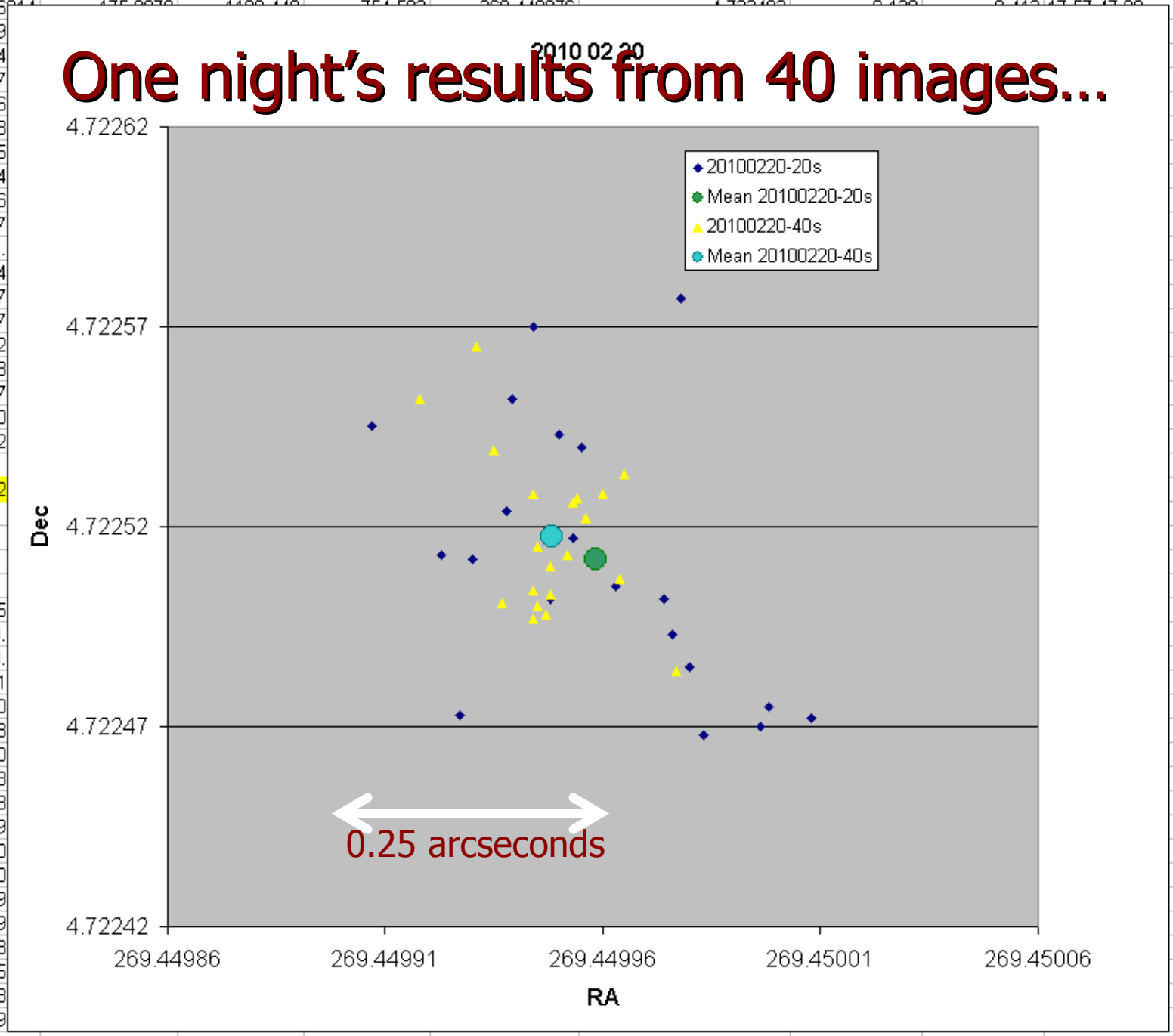
Tools Data Window Help Adobe PDF

100% Arial 10

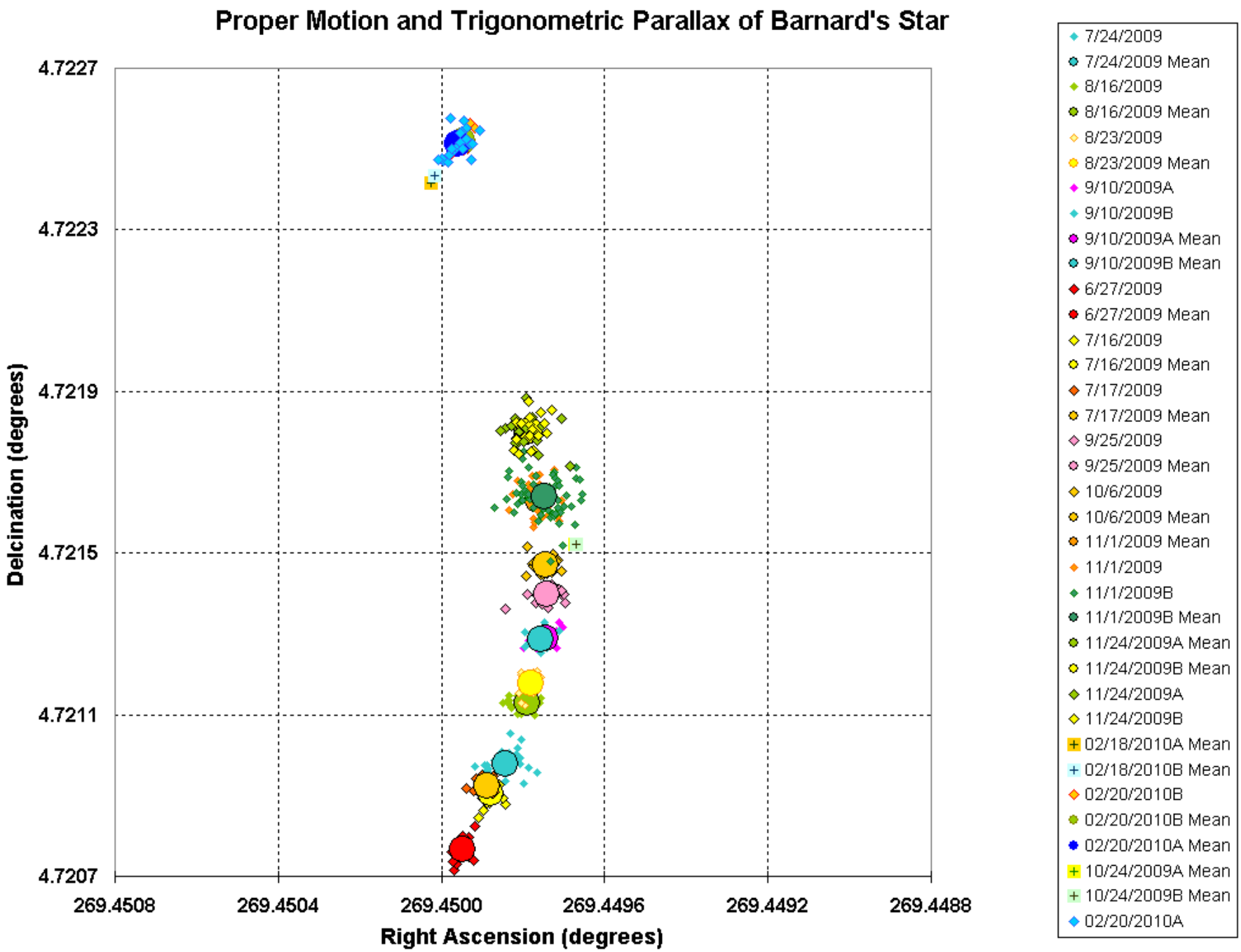
Check the "canaries"...

	C	D	E	F	G	H	I
Format							
	Focal[mm]	PA[d.d]	Xtarget	Ytarget	RA(decimal)	DEC(decimal)	Rarms
2455249.05	910.7577	176.0292	1108.009	755.57	269.44993	4.722512	0.206
2455249.051	910.6914	175.9878	1108.448	754.582	269.449976	4.722493	0.129
2455249.052	910.9029	175.9701	1108.29	754.921	269.449963	4.722505	0.168
2455249.054	911.4508	175.973	1108.361	754.588	269.449996	4.72247	0.25
2455249.055	911.7439	175.9687	1108.577	756.159	269.449939	4.722552	0.189
2455249.056	910.6556	175.9618	1108.565	755.241	269.449907	4.722545	0.27
2455249.057	910.8012	175.9552	1108.531	754.767	269.449983	4.722468	0.25
2455249.059	910.5681	176.0015	1108.018	754.967	269.449923	4.722513	0.24
2455249.06	910.4368	175.9702	1108.287	754.621	269.449978	4.722577	0.148
2455249.061	910.6304	175.9787	1107.618	754.792	269.44998	4.722485	0.18
2455249.062	910.7607	175.9954	1108.215	755.205	269.449944	4.72257	0.219
2455249.063	911.651	175.9898	1107.924	755.604	269.449998	4.722475	0.313
2455249.065	911.4264	176.0058	1108.208	754.577	269.44995	4.722543	0.151
2455249.066	910.7596	176.0108	1108.338	755.44	269.449953	4.722517	0.202
2455249.067	912.7834	176.0043	1108.74	755.447	269.449927	4.722473	0.389
2455249.068	911.2181	176.0083	1108.578	755.38	269.449938	4.722524	0.185
2455249.07	910.3985	175.9885	1109.107	755.217	269.449974	4.722502	0.263
2455249.071	910.7937	176.0167	1109.085	754.485	269.449948	4.722502	0.232
2455249.072	910.0056	175.9565	1109.656	754.628	269.450008	4.722472	0.364
2455249.073	910.2935	175.9888	1109.667	754.114	269.449955	4.72254	0.164
					269.4499585	4.7225119	
	0.627392588	0.020966727			2.75098E-05	3.3812E-05	0.069730609
					0.099035305	0.121723269	
asurement Tool							
Format							
	Focal[mm]	PA[d.d]	Xtarget	Ytarget	RA(decimal)	DEC(decimal)	Rarms
2455249.051	910.5968	175.9881	1108.076	754.793	269.449947	4.722498	0.143
2455249.052	910.603	175.9753	1108.661	754.43	269.449944	4.722528	0.191
2455249.053	910.655	175.9717	1108.461	754.861	269.449952	4.722513	0.189

One night's results from 40 images...



910.6	176.0292	1108.009	755.57	269.44993	4.722512	0.206	0.266	17 57 47.98	+04 43 21.0
910.6									+04 43 21.0
910.9									+04 43 21.0
911.4									+04 43 20.9
911.7									+04 43 21.2
910.6									+04 43 21.2
910.8									+04 43 20.9
910.5									+04 43 21.0
910.4									+04 43 21.3
910.6									+04 43 20.9
910.7									+04 43 21.3
911.									+04 43 20.9
911.4									+04 43 21.2
910.7									+04 43 21.1
912.7									+04 43 20.9
911.2									+04 43 21.1
910.3									+04 43 21.0
910.7									+04 43 21.0
910.0									+04 43 20.9
910.2									+04 43 21.1
0.627392									
nt Tool									
Focal[mm]									DEC(ddmmss)
910.5									+04 43 21.0
910.									+04 43 21.1
910.									+04 43 21.0
911.1									+04 43 21.0
911.0									+04 43 21.1
910.8									+04 43 21.0
911.0									+04 43 21.1
910.8									+04 43 21.0
911.3									+04 43 21.1
910.9									+04 43 21.0
911.0									+04 43 21.0
911.0									+04 43 21.1
910.9									+04 43 20.9
910.9									+04 43 21.1
911.3									+04 43 21.0
910.5									+04 43 21.1
910.8									+04 43 21.1
910.9									+04 43 21.2
911.0697	175.9907	1109.513	753.353	269.449948	4.722503	0.13	0.206	17 57 47.99	+04 43 21.0


Proper Motion and Trigonometric Parallax of Barnard's Star
4.7227
4.7223
4.7219
4.7215
4.7211
4.7207
Delcination (degrees)
269.4508
269.4504
269.4500
269.4496
269.4492
269.4488
Right Ascension (degrees)
7/24/2009
7/24/2009 Mean
8/16/2009
8/16/2009 Mean
8/23/2009
8/23/2009 Mean
9/10/2009A
9/10/2009B
9/10/2009A Mean
9/10/2009B Mean
6/27/2009
6/27/2009 Mean
7/16/2009
7/16/2009 Mean
7/17/2009
7/17/2009 Mean
9/25/2009
9/25/2009 Mean
10/6/2009
10/6/2009 Mean
11/1/2009 Mean
11/1/2009
11/1/2009B
11/1/2009B Mean
11/24/2009A Mean
11/24/2009B Mean
11/24/2009A
11/24/2009B
02/18/2010A Mean
02/18/2010B Mean
02/20/2010B
02/20/2010B Mean
02/20/2010A Mean
10/24/2009A Mean
10/24/2009B Mean
02/20/2010A

The Next Steps…

- Model based on five parameters:
  - Initial RA (J 2000.0)
  - Initial DEC (J 2000.0)
  - PM in RA
  - PM in DEC
  - Parallax
  - These known from Hipparcos Mission
- Compute parameters from observations
  - Solve matrix of observed (RA,DEC).
  - Least-squares method for best fit to observations.

Computing a star's position…

$$\alpha_{now} = \alpha_{J2000.0} + \alpha_{PM}(Y_{now} - 2000) + \pi P_{\alpha}$$
$$\delta_{now} = \delta_{J2000.0} + \delta_{PM}(Y_{now} - 2000) + \pi P_{\delta}$$

- $(\alpha, \delta)_{now}$ = current coordinates
- $(\alpha, \delta)_{J2000.0}$ = coordinates in J2000.0
- α_{PM} = annual proper motion in RA
- δ_{PM} = annual proper motion in Dec
- π = parallax of the star
- P_{α} = parallax factor in α for time Y_{now}
- P_{δ} = parallax factor in δ for time Y_{now}

Trigonometric Parallax and Proper Motion

Setup (J2000.0 = JDE 2451545.0)

Initial RA [deg]:	269.45206
Initial Dec [deg]:	4.69344
Trig Parallax ["]:	0.5454
PM in RA ["/y]:	-0.79871
PM in Dec ["/y]:	10.33777

Time Span

YYYY-MM-DD:	2009-06-01
Step Size [d]:	10
YYYY-MM-DD:	2009-12-31

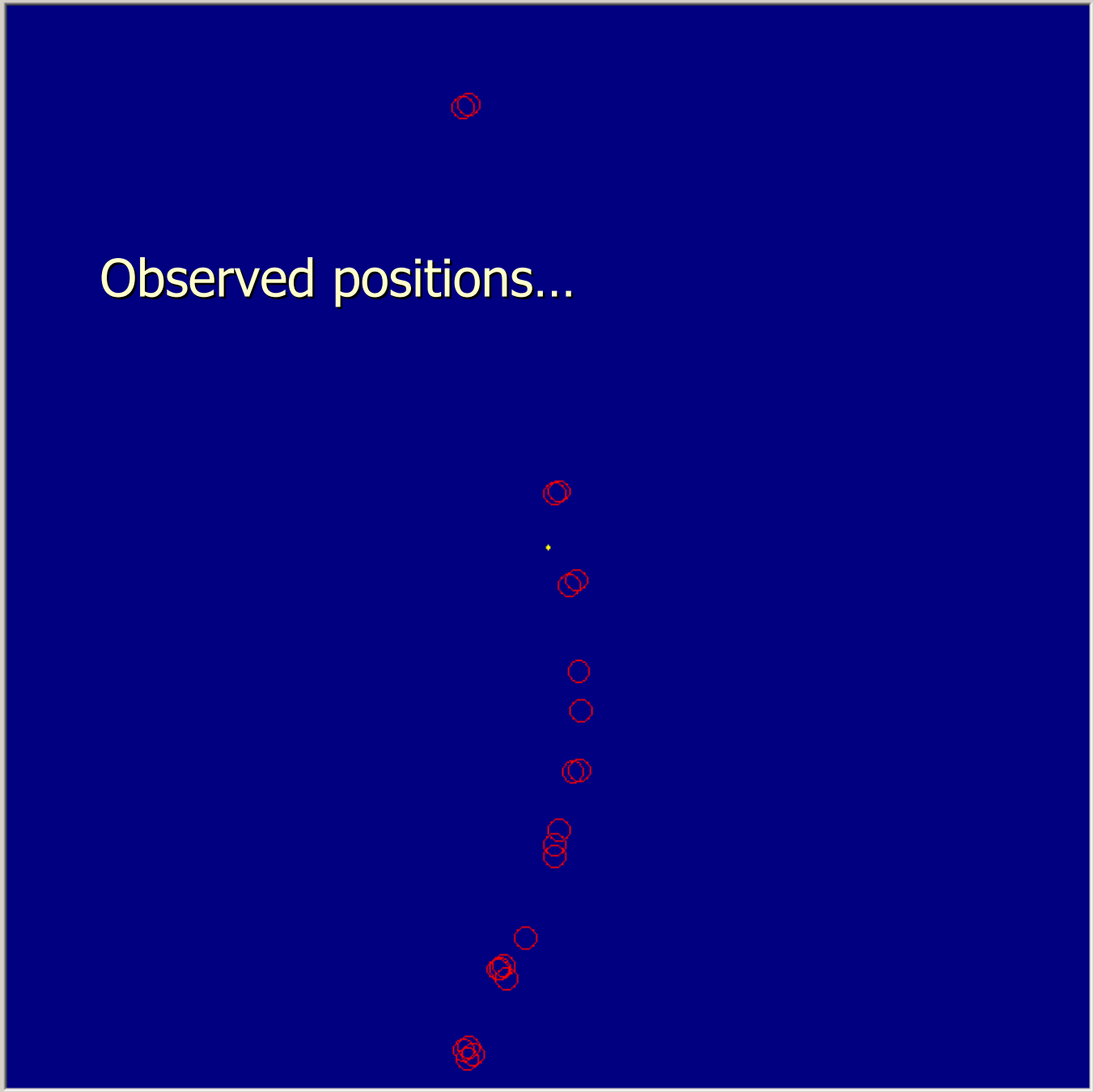
Graph Properties

Center RA [d]:	269.4498
Center Dec [d]:	4.7217
Size [d]:	0.0020

Task

- ( ) Plot positions for "Time-Span" times
- (•) Plot observed positions
- ( ) Plot positions for observed times
- ( ) Display observed times and positions

Execute Task



Trigonometric Parallax and Proper Motion

Setup (J2000.0 = JDE 2451545.0)

Initial RA [deg]:	269.45206
Initial Dec [deg]:	4.69344
Trig Parallax ["]:	0.5454
PM in RA ["/y]:	-0.79871
PM in Dec ["/y]:	10.33777

Time Span

YYYY-MM-DD:	2009-06-01
Step Size [d]:	10
YYYY-MM-DD:	2009-12-31

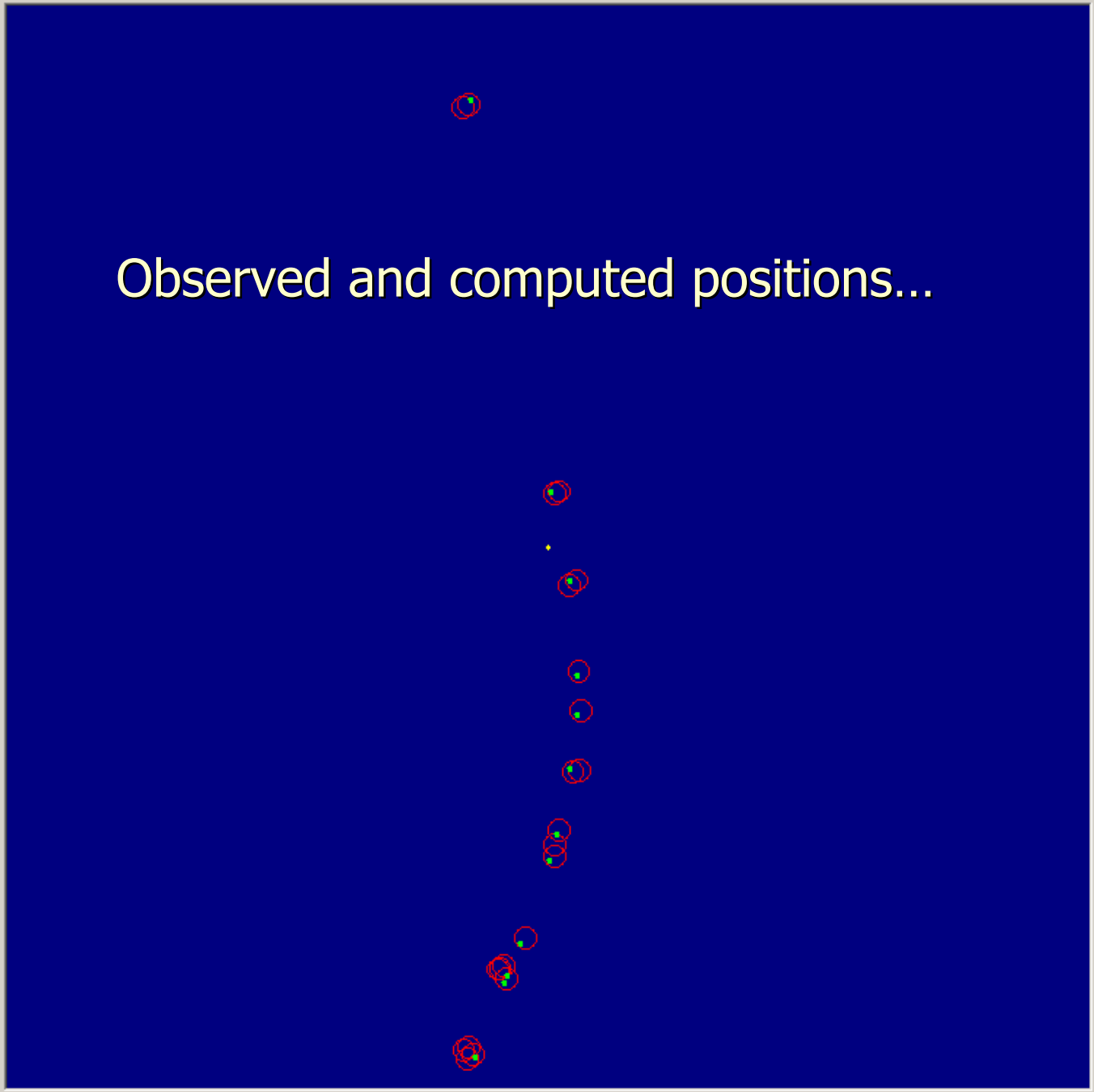
Graph Properties

Center RA [d]:	269.4498
Center Dec [d]:	4.7217
Size [d]:	0.0020

Task

- ○ Plot positions for "Time-Span" times
- ○ Plot observed positions
- ◉ Plot positions for observed times
- ○ Display observed times and positions

Execute Task



Trigonometric Parallax and Proper Motion

Setup (J2000.0 = JDE 2451545.0)

Initial RA [deg]:	269.4498
Initial Dec [deg]:	4.7217
Trig Parallax ["]:	0.5454
PM in RA ["/y]:	0
PM in Dec ["/y]:	0

Time Span

YYYY-MM-DD:	2009-06-01
Step Size [d]:	7
YYYY-MM-DD:	2010-06-01

Graph Properties

Center RA [d]:	269.4498
Center Dec [d]:	4.7217
Size [d]:	0.0020

Task

- (•) Plot positions for "Time-Span" times
- ( ) Plot observed positions
- ( ) Plot positions for observed times
- ( ) Display observed times and positions

Execute Task

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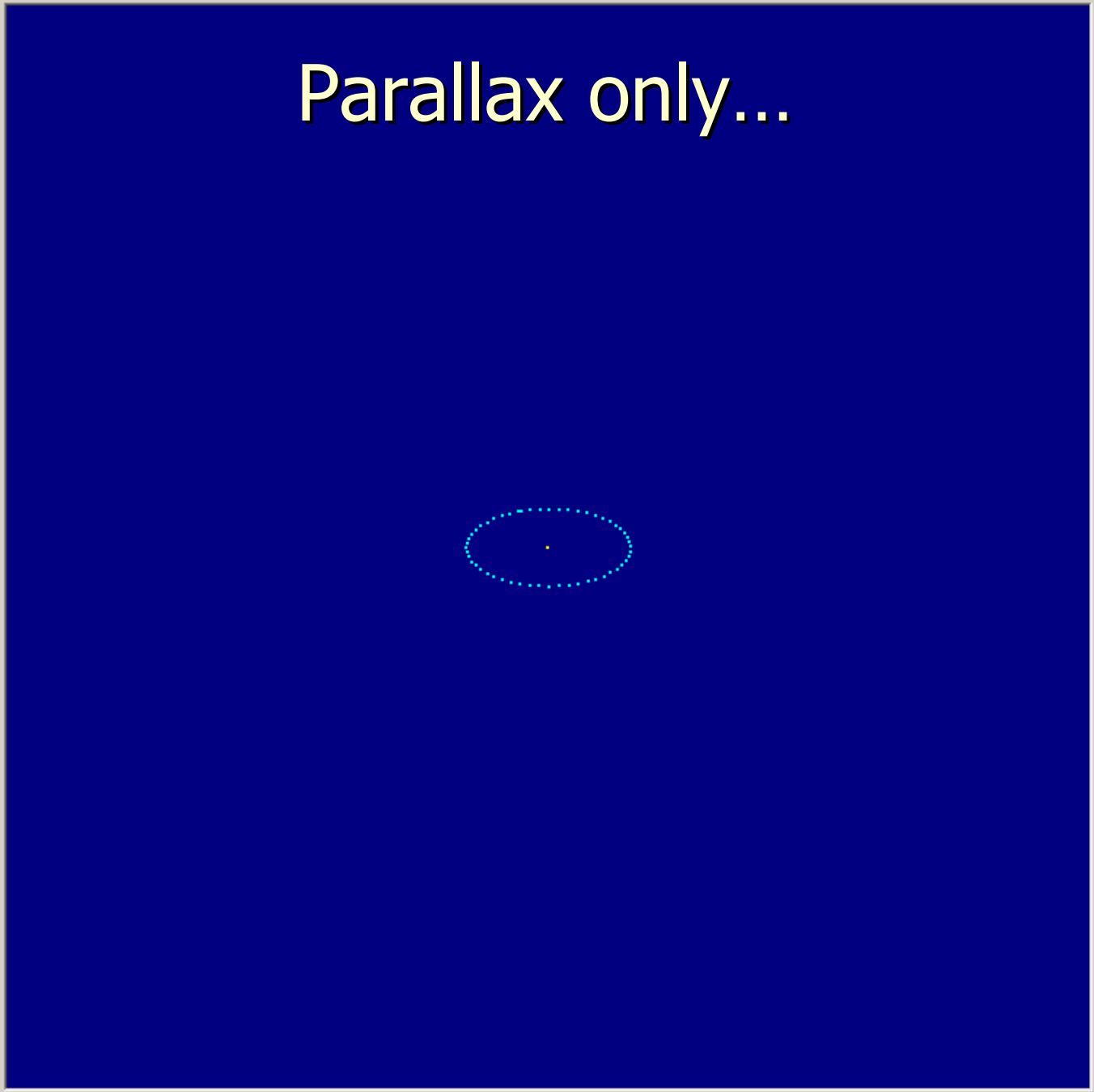
JD= 2455347.5
PFalpha= 0.3561
PFdelta= 0.4468
RA= 269.4499
Dec= 4.7218
```



Trigonometric Parallax and Proper Motion

Setup (J2000.0 = JDE 2451545.0)

Initial RA [deg]:	269.45206
Initial Dec [deg]:	4.69344
Trig Parallax ["]:	0
PM in RA ["/y]:	-0.79871
PM in Dec ["/y]:	10.33777

Time Span

YYYY-MM-DD:	2009-06-01
Step Size [d]:	7
YYYY-MM-DD:	2010-06-01

Graph Properties

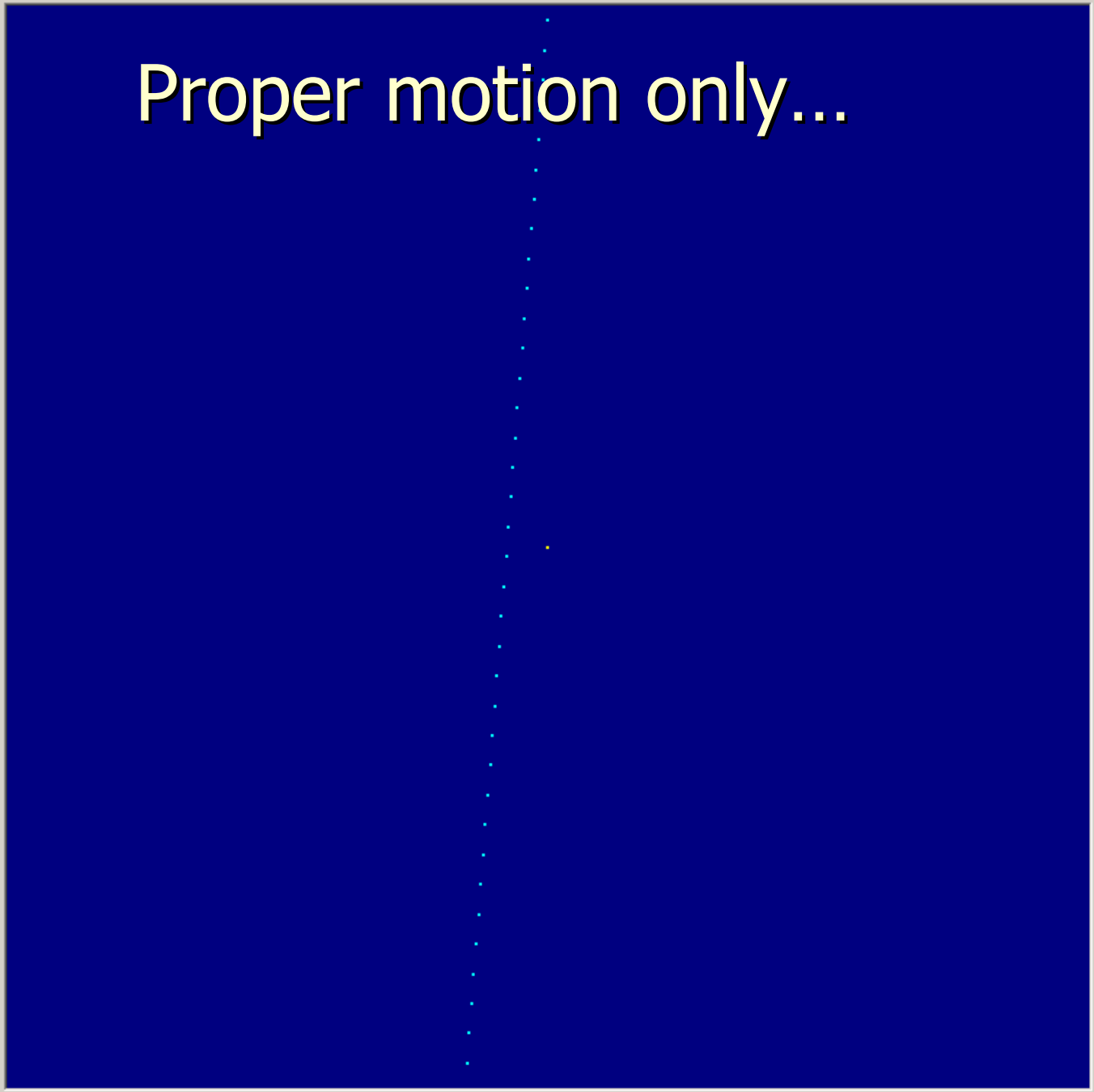
Center RA [d]:	269.4498
Center Dec [d]:	4.7217
Size [d]:	0.0020

Task

- (•) Plot positions for "Time-Span" times
- ( ) Plot observed positions
- ( ) Plot positions for observed times
- ( ) Display observed times and positions

Execute Task

JD= 2455347.5
PFalpha= 0.3561
PFdelta= 0.4464
RA= 269.4498
Dec= 4.7233



Trigonometric Parallax and Proper Motion

Setup (J2000.0 = JDE 2451545.0)

Initial RA [deg]:	269.45206
Initial Dec [deg]:	4.69344
Trig Parallax ["]:	0.5454
PM in RA ["/y]:	-0.79871
PM in Dec ["/y]:	10.33777

Time Span

YYYY-MM-DD:	2008-06-01
Step Size [d]:	10
YYYY-MM-DD:	2011-06-01

Graph Properties

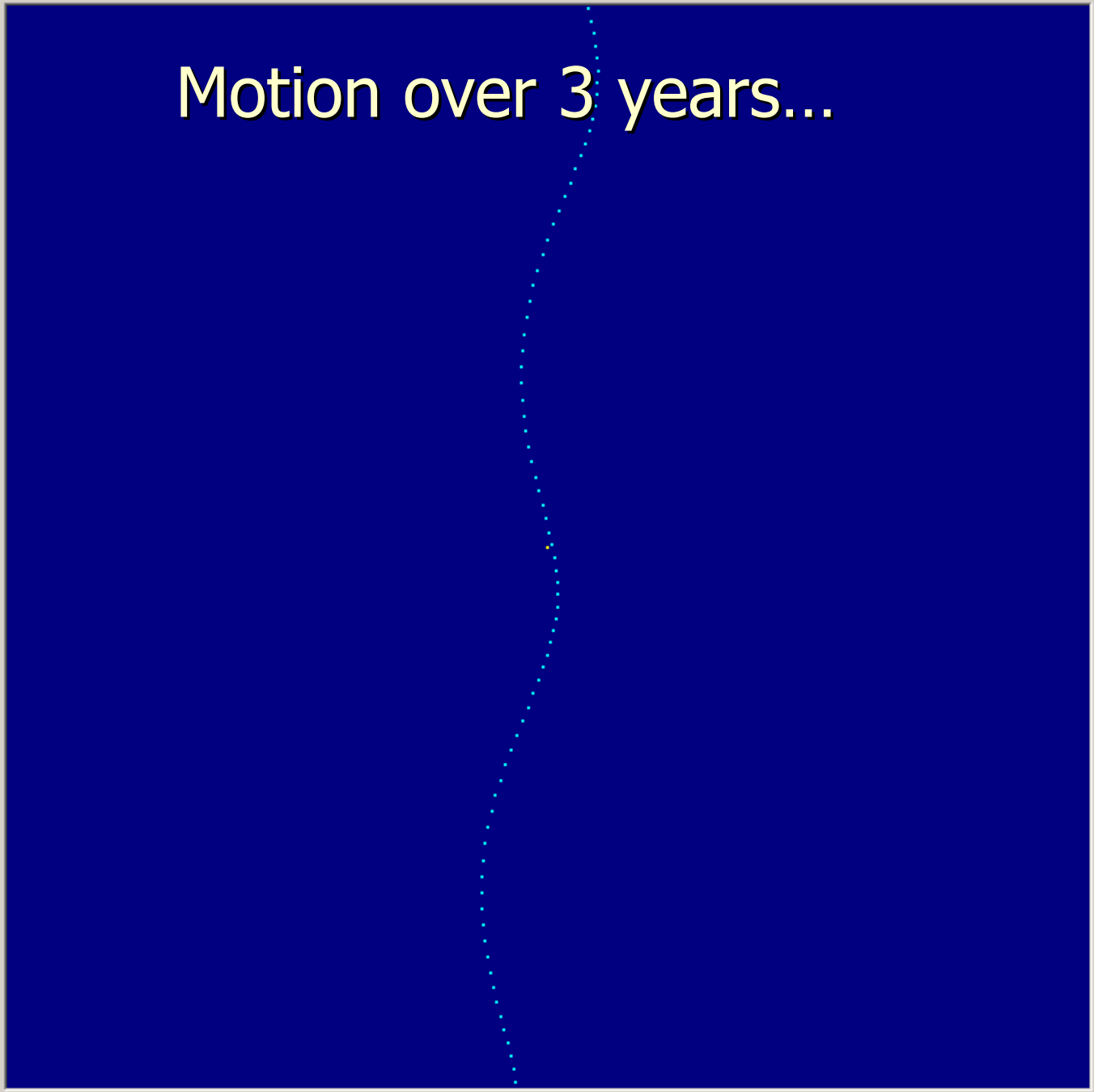
Center RA [d]:	269.4498
Center Dec [d]:	4.7217
Size [d]:	0.0060

Task

- (•) Plot positions for "Time-Span" times
- ( ) Plot observed positions
- ( ) Plot positions for observed times
- ( ) Display observed times and positions

Execute Task

JD= 2455708.5
PFalpha= 0.4231
PFdelta= 0.4326
RA= 269.4496
Dec= 4.7262



Trigonometric Parallax and Proper Motion

Setup (J2000.0 = JDE 2451545.0)

Initial RA [deg]:	269.45206
Initial Dec [deg]:	4.69344
Trig Parallax ["]:	0.5454
PM in RA ["/y]:	-0.79871
PM in Dec ["/y]:	10.33777

Time Span

YYYY-MM-DD:	2008-06-01
Step Size [d]:	5
YYYY-MM-DD:	2011-06-01

Graph Properties

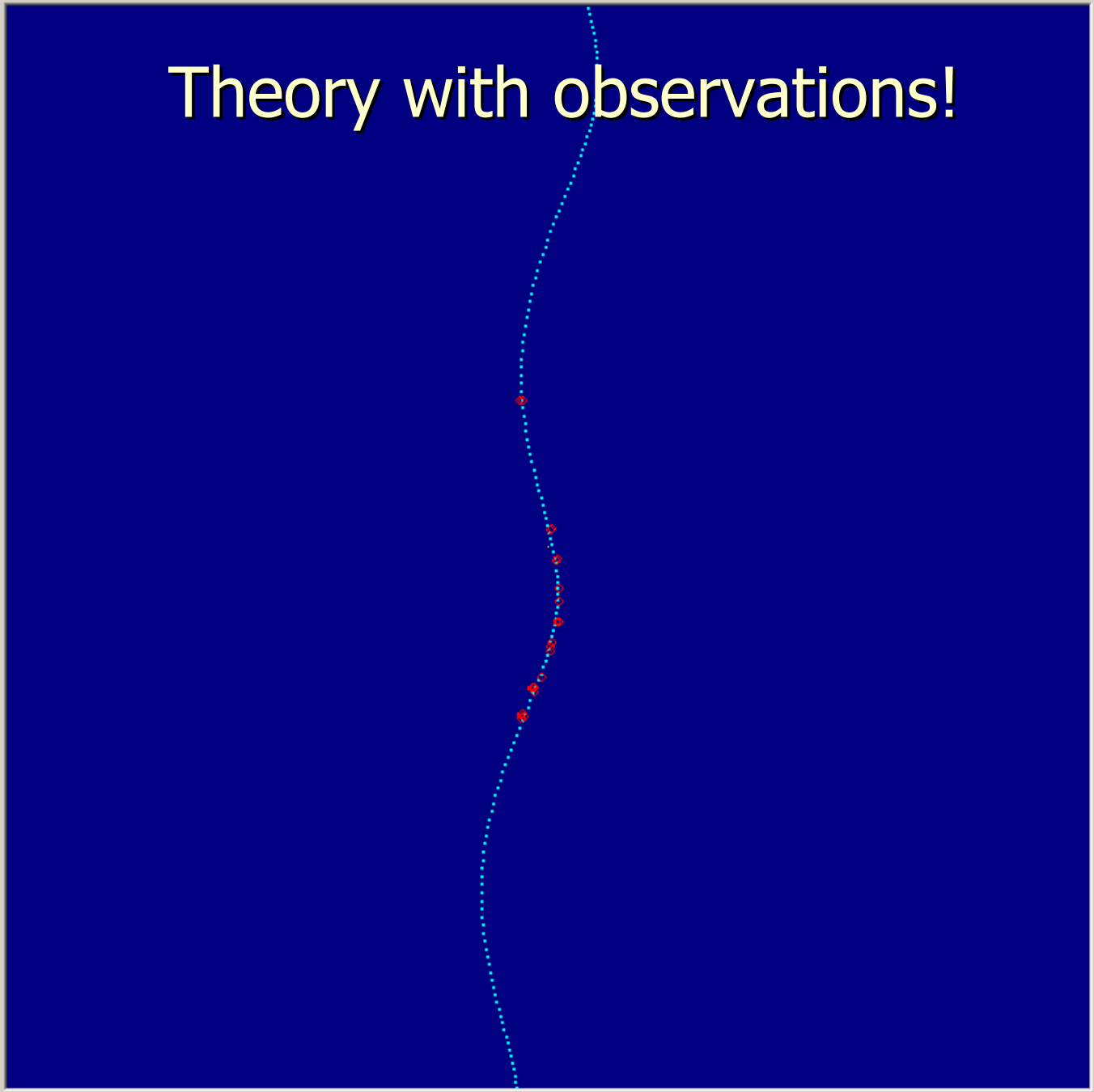
Center RA [d]:	269.4498
Center Dec [d]:	4.7217
Size [d]:	0.0060

Task

- ( ) Plot positions for "Time-Span" times
- (•) Plot observed positions
- ( ) Plot positions for observed times
- ( ) Display observed times and positions

Execute Task

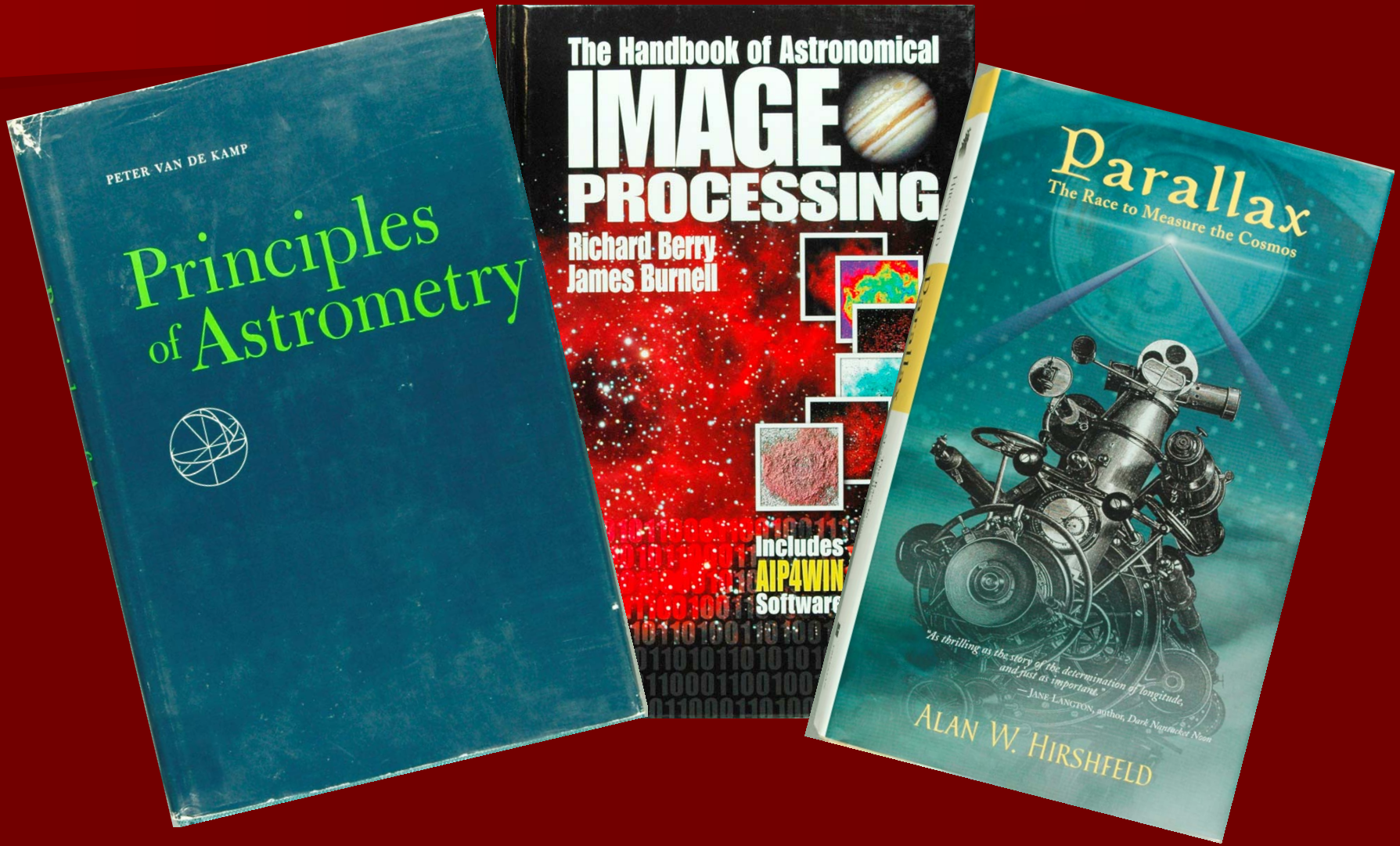
JD= 2455713.5
PFalpha= 0.3450
PFdelta= 0.4484
RA= 269.4496
Dec= 4.7262



Small-Telescope Astrometry

- With a focal length ~1,000mm.
- Ordinary CCD with 6.4 micron pixels.
- Selected set of reference stars.
- Observation with multiple images.
- Using optimized exposure time.
- Routinely achieves 0.020 arcsecond accuracy.
- Sometimes achieves 0.010 arcsecond accuracy.

Resources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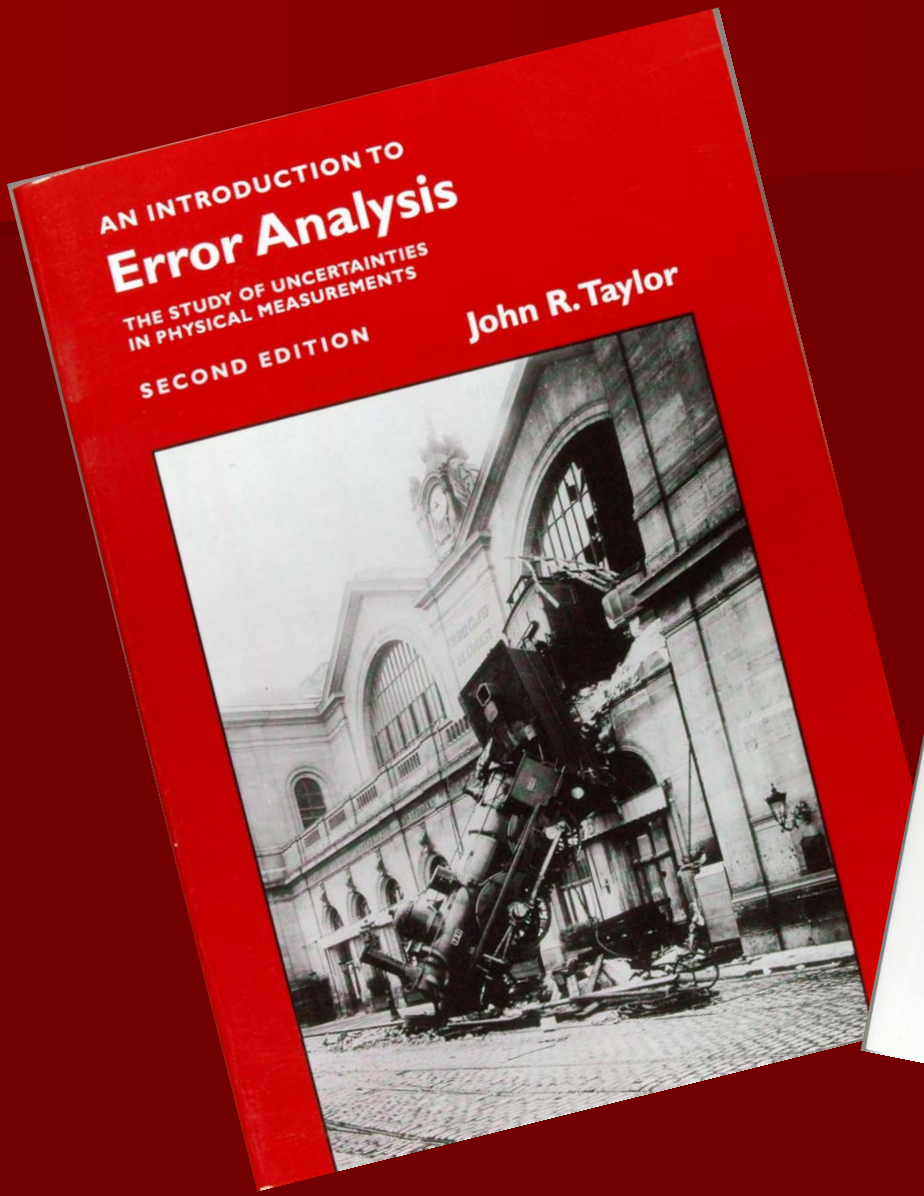
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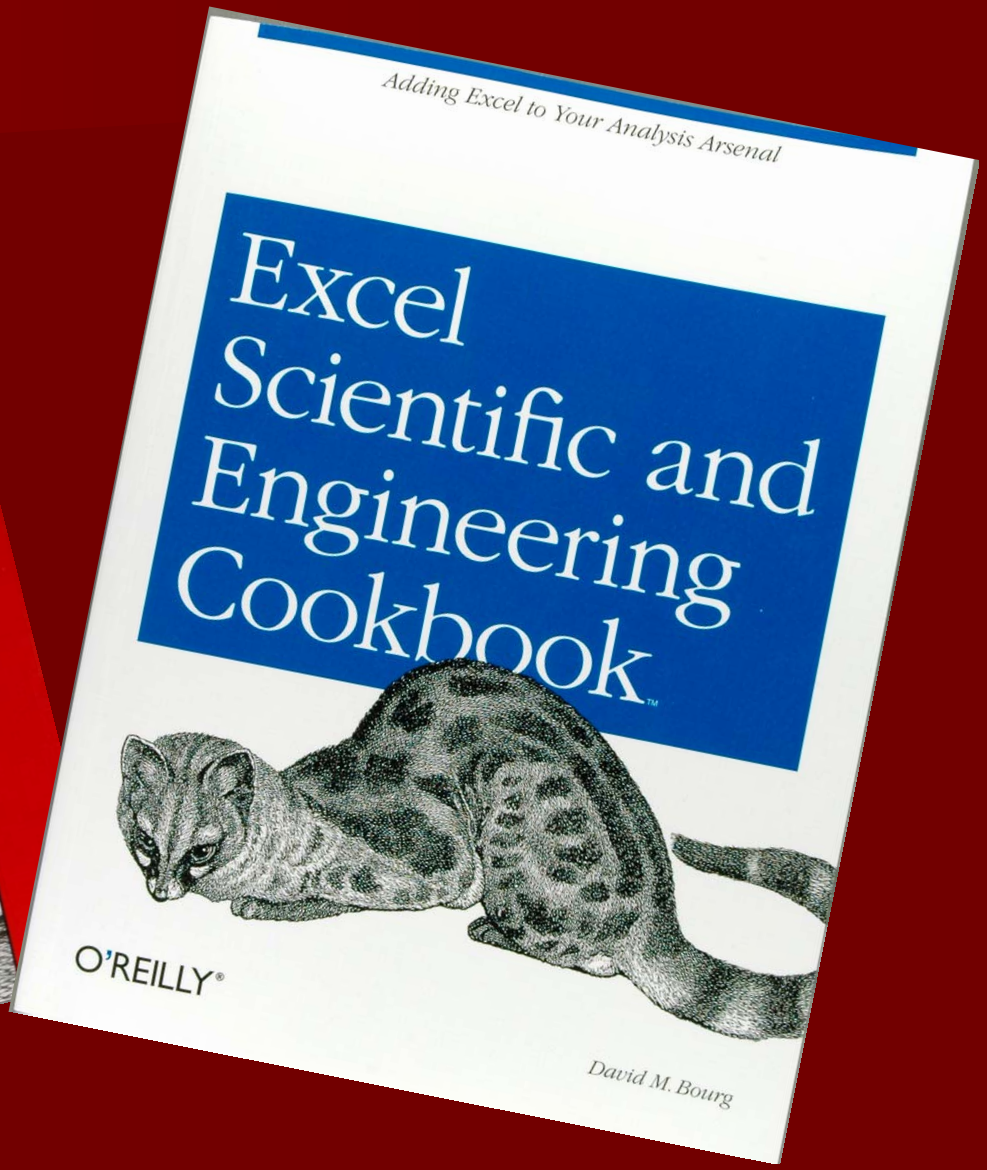
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Thank You!

Richard Berry